JOINT INSTITUTE FOR NUCLEAR RESEARCH
Veksler and Baldin laboratory of High Energy Physics

FINAL REPORT ON THE INTEREST PROGRAMME

Puzzles of Multiplicity

Supervisor:
Prof. Elena Kokoulina

Student:
Egor Filin, Russia,
MIPT

Participation period:
March 03 – April 20, Wave 12

Dubna, 2025

Contents

1 Abstract

Describing the quark-gluon cascade, we come to the necessity of using Markov branching processes. The process of hadronization is considered using the example of electron-positron annihilation. The distribution by the number of charged particles is described. Based on experimental data, a fitting is made to find the distribution parameters.

2 Introduction

One of the observables in the high energy experiment is multiplicity - the number of secondary particles. The most popular one is multiplicity of charged particles. Experimenters also restore multiplicity of neutral particles. The main statistical characteristics of multiplicity are its mean value and a variance. pQCD allows us to calculate hard processes but it has a hard time with description of the hadronization stage. Two stage model offers to add to the quark-gluon stage the phenomenological hadronization stage and calculate multiplicity distribution for electron-positron annihilation. The comparison of this model with data demonstrates good agreement.

3 Main part

3.1

Based on Giovannini's idea, we consider the quark-gluon jet as a Markov branching process. Y is introduced as an evolutionary parameter.

$$Y = \frac{1}{2\pi b} \ln\left(1 + \alpha b \ln\frac{Q^2}{\mu^2}\right)$$

The main possible events are:

1. Gluon fission
2. Quark bremsstrahlung (i.e. emission of a gluon by a quark)
3. Creation of a quark-antiquark pair from a gluon.

The expressions for the probability of each of these events are, respectively.

1. $P = A\Delta Y$
2. $P = \tilde{A}\Delta Y$
3. $P = B\Delta Y$

where A, $\tilde{A}$, B are constants that do not depend on Y.

The probability for a gluon or a quark to convert into m_q quarks and m_g gluons in the interval $(Y, Y+\Delta Y)$ is given by the following sum of probabilities:

For gluon:

$$\delta_{1m_g}\delta_{0m_g} + a_{m_g m_q}\Delta Y = 1$$

Taking into account only the above events, we get

$$1 + (a_{10} + a_{20} + a_{02})\Delta Y = 1$$

$$a_{10} = -(a_{20} + a_{02})$$

And for quark:

$$\delta_{0m_g}\delta_{1m_g} + a_{m_g m_q}\Delta Y = 1$$

$$1 + (a_{11} + a_{01})\Delta Y = 1$$

$$a_{01} = -a_{11}$$

The generating function acting on the interval ΔY has the form:

$$w(u_g, u_q) = \sum_{m_g, m_q = 0}^{\infty} a_{m_g, m_q} u_g^{m_g} u_q^{m_q}$$

Let's define $a_{20} = A$, $a_{02} = B$, $a_{11} = \tilde{A}$. Then the generating functions for gluon and quark will be:

$$w^g(u_g, u_q) = (-A - B)u_g + Au_g^2 + Bu_q^2$$

$$w^q(u_g, u_q) = -\tilde{A}u_q + \tilde{A}u_q u_g$$

3.2

For process $(m_g, m_q) \to (n_g, n_q)$ we introduce the probability $P_{m_g m_q n_g n_q}(Y)$, then the generating functions for gluon and quark will be:

$$\sum_{n_g, n_q = 0}^{\infty} P_{1,0,n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = G(u_g, u_q, Y)$$

$$\sum_{n_g, n_q = 0}^{\infty} P_{0,1,n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = Q(u_g, u_q, Y)$$

Due to the independent action of the individual partons it can be shown as:

$$\sum_{n_g, n_q = 0}^{\infty} P_{m_g,m_q,n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = Q^{m_g}(u_g, u_q, Y) G^{m_q}(u_g, u_q, Y)$$

Considering this expression at the moment $Y + \Delta Y$ we obtain

$$G(u_g, u_q, Y + \Delta Y) = G(u_g + w^g \Delta Y, u_q + w^q \Delta Y, Y)$$

$$\frac{\partial G}{\partial Y} = \frac{\partial G}{\partial u_g} w^g + \frac{\partial G}{\partial u_q} w^q$$

Similarly, for the quark generating function Q and using the expressions for w^g and w^q we obtain a system of differential equations

$$\begin{cases} \frac{\partial G}{\partial Y} = -(A+B)G + AG^2 + BQ^2 \\ \frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \end{cases}$$

The probability of one gluon to form a state (n_g, n_q) on an interval $Y + \Delta Y$ is:

$$\begin{aligned} P_{1,0,n_g,n_q}(Y+\Delta Y) = {} & \left[1 - \tilde{A} n_q \Delta Y - A n_g \Delta Y - B n_g \Delta Y\right] P_{1,0,n_g,n_q}(Y) \\ & + \tilde{A} n_q \Delta Y \, P_{1,0,n_g-1,n_q}(Y) + A(n_g-1)\Delta Y \, P_{1,0,n_g-1,n_q}(Y) \\ & + B(n_g+1)\Delta Y \, P_{1,0,n_g+1,n_q-2}(Y) + o(\Delta Y) \end{aligned}$$

Also acting with the quark probability, and also considering the creation of a quark-antiquark pair from a gluon process to be suppressed in comparison with the other two processes under consideration, we obtain this differential equations and their solutions:

The gluon jet:

$$\frac{dP_m}{dY} = -AmP_m + A(m-1)P_{m-1}$$

solution:

$$P_m = e^{-AY}(1 - e^{-AY})^{m-1}$$

And the corresponding generating function

$$G = \frac{u_g e^{-AY}}{1 - u_g(1 - e^{-AY})}$$

The quark jet:

$$\frac{dP_m}{dY} = -\tilde{A}P_m - AmP_m + \tilde{A}P_{m-1} + A(m-1)P_{m-1}$$

solution:

$$P_m = \frac{\mu(\mu+1)\cdots(\mu+m-1)}{m!} e^{-\tilde{A}Y} \left(1 - e^{-AY}\right)^m$$

where $\mu = \frac{\tilde{A}}{A}$. The corresponding generating function

$$Q = u_q \left(\frac{e^{-AY}}{1 - u_g(1 - e^{-AY})}\right)^\mu$$

Probability for convenience we will write as follows

$$P_m = \frac{\mu(\mu+1)\cdots(\mu+m-1)}{m!} \left(\frac{\mu}{\mu + \overline{m}}\right)^\mu \left(\frac{\overline{m}}{\overline{m} + \mu}\right)^m$$

It is negative binomial distribution.

3.3

Let's consider the process of electron-positron annihilation. In the first stage, through an intermediate photon or Z-boson, a quark-antiquark pair is formed.

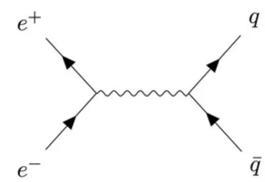

Figure 1: First stage

The pair begins to emit gluons, thus forming a quark-gluon cascade. In the second stage, the predominant particles are gluons. And the third stage is the formation of hadrons, in which some of the gluons, not having hadronized, will remain and will create mass.

The hadronization generating function for gluon or quark has the form

$$Q^H(z) = \left(1 - \frac{\overline{n}^H}{N} + \frac{\overline{n}^H}{N} z\right)^N$$

where N and $\overline{n}$ are the maximum and the average number of hadrons that can be produced from a gluon or a quark.

Let quarks and gluons have the same hadronization probability, then

$$\frac{\overline{n}_g^H}{N_g} \approx \frac{\overline{n}_q^H}{N_q} => \frac{\overline{n}_g^H}{\overline{n}_q^H} = \frac{N_g}{N_q} = \alpha$$

Then the result distribution will be:

$$P_n = \Omega \sum P_m [Q^H]^{(2+\alpha m)N}$$

where Ω is the normalization factor

$$P_n = \Omega \left(\frac{\mu}{\mu + \overline{m}}\right)^\mu \left[C_{2N}^n \left(\frac{\overline{n}^H}{N}\right)^n \left(1 - \frac{\overline{n}^H}{N}\right)^{2N-n} + \right.$$
$$+ \sum_{m=1}^{M} \frac{\mu(\mu+1)\cdots(\mu+m-1)}{m!} \left(\frac{\overline{m}}{\overline{m}+\mu}\right)^m C_{(2+\alpha m)N}^n$$
$$\left. \times \left(\frac{\overline{n}^H}{N}\right)^n \left(1 - \frac{\overline{n}^H}{N}\right)^{(2+\alpha m)N-n} \right]$$

4 Conclusion

The resulting expression for the multiplicity distribution was fitted to the experimental data at an energy of 22 GeV and the following results were obtained

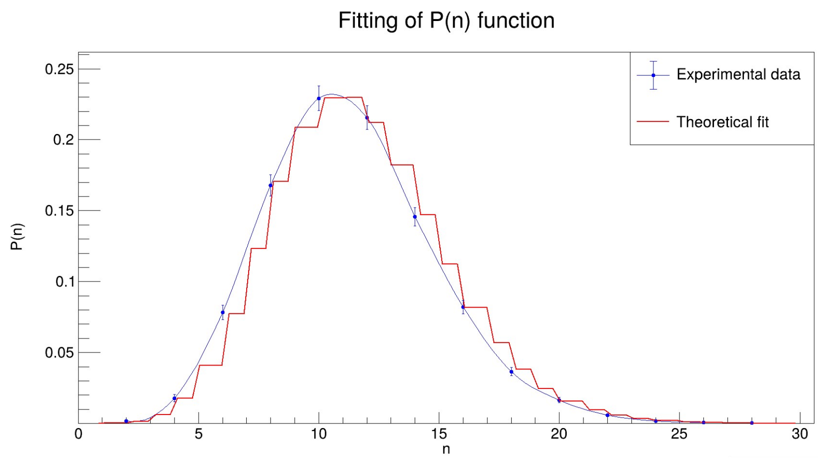

Figure 2: Graph

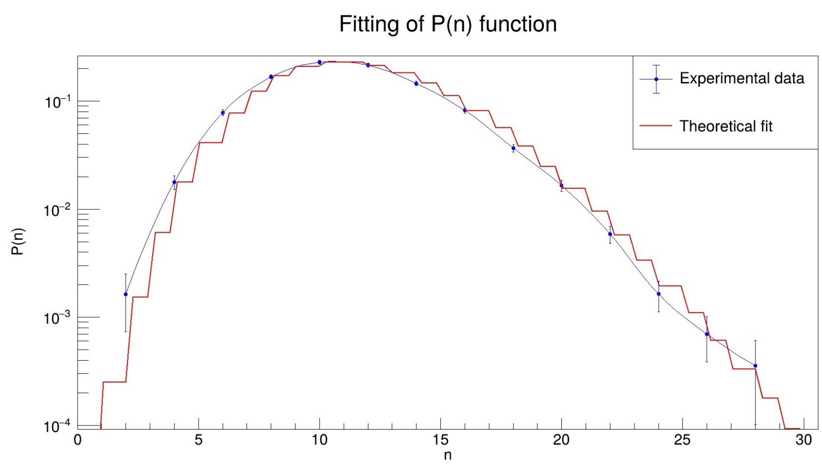

Figure 3: Graph in logarithmic scale

```
Minimizer is Minuit2 / Migrad
Chi2                      =      1.86645
NDf                       =            8
Edm                       =  1.49942e-07
NCalls                    =         1230
mu                        =      7.05847   +/-   3.62088
m_mean                    =      4.57473   +/-   1.0474
n^h                       =      3.91531   +/-   0.287111
N                         =      13.0001   +/-   6.96805e-05
alpha                     =          0.2   +/-   2.38072e-07
Omega                     =      2.30147   +/-   0.0501263
```

Figure 4: Parameters

5 Acknowledgments

I express my sincere gratitude to E. S. Kokoulina for her help in mastering new material and comprehensive support.

6 References

1. Giovannini A. Nucl.Phys. B161(1979)429.

2. Dokshitzer, Yu.L. Hard semi-inclusive processes in QCD. Physics Letters B. – 1978. – Vol. 78. – P. 290-294.

3. Müller, B. (2005). Hadronic signals of deconfinement at RHIC. Nuclear Physics A, 750(1), 84–97.

4. M. Althoff et al.: Jet Production and Fragmentation in e^+e^- Annihilation. Z. Phys. C - Particles and Fields 22, 307-340 (1984).

5. Kuvshinov V.I., Kokoulina E.S. Acta Phys.Pol.B13 (1982) 533.

6. Kokoulina E.S., Kuvshinov V.I. Vesti AN BSSR. Ser.Phys.-Math.Sc.3(1988)49.

7. Kokoulina E.S. Proc. Sem. NPCS (1991,1995,1999).