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FINAL REPORT ON THE INTEREST PROGRAMME

Numerical simulations for magnetic moment dynamics in Josephson junction coupled to a nanomagnet

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CONTENTS

Abstract

The study investigates the dynamics of a Josephson junction coupled to nanomagnet system, focusing on the reorientation of the magnetic moment and the influence of model parameters such as the Josephson energy-to-magnetic energy ratio G and the Josephson frequency ω_j. The study employs numerical simulations based on the Landau-Lifshitz-Gilbert (LLG) equation to analyze the systems behavior under varying the model parameters. Special attention is given to the role of initial conditions, the stability of dynamical points, and the interplay between superconducting and magnetic components in determining the systems evolution.

A detailed analysis of stable and unstable points shows that an increase in G and ω_j leads to dynamic stabilization and a complete reorientation of the magnetic moment easy axis. This phenomenon is explained through the Kapitza method, which highlights the stabilizing effect of high-frequency driving forces. Additionally, the study explores how the role of the effective field of the quasiparticle can affect the frequency dependance of the reorientation of the easy-axis.

The results demonstrate the potential for controlling nanomagnet dynamics using Josephson junctions, with implications for spintronics and quantum computing applications. By providing insights into the mechanisms governing magnetic moment reorientation, this work contributes to the development of novel devices that leverage hybrid superconducting-magnetic systems for advanced functionalities.

Introduction

The study of superconductor-based heterostructures is one of the most dynamically developing areas of modern condensed matter physics. Systems combining superconducting and other various components are of particular interest, which makes it possible to study mutual influence and realize new states.

Josephson junctions (JJ) occupy a central place in the study of such hybrid structures. Traditional SIS junctions (superconductor-insulator-superconductor) have evolved into more complex configurations, such as SFS junctions (superconductor-ferromagnetic-superconductor) and φ_0 junctions, demonstrating unique coupling properties between superconducting current and magnetic moment.

Mathematical modeling of resonant phenomena in such systems often uses a mechanical analogy with the classical Kapitsa pendulum. This model helps to understand the mechanisms of occurrence of resonances in systems with external periodic effects. Unique resonant phenomena are observed due to the interaction between the magnetic and superconducting subsystems. In particular, in the Josephson junction – nanomagnetic system, the analogy with the Kapitsa pendulum is manifested through the change of the position of stable equilibrium of the magnetic moment when the ratio between the Josephson and magnetic energies changes. Various numerical methods are used for modeling the magnetization dynamics in such systems. The fundamental tool remains the Landau-Lifshitz-Gilbert equation (LLG), proposed in 1935 and continuously improved since then. This equation makes it possible to effectively describe a wide range of phenomena, from the simplest fluctuations of the magnetic moment to complex nonlinear effects and chaotic dynamics.

In the field of studying SFS types of the Josephson junction, special attention is paid to the study of resonance phenomena and bifurcation processes in hybrid magnetic-superconducting structures, which can become the basis for the creation of new types of information processing devices.

1. Model

We consider a short Josephson junction (JJ) with length l, coupled to a single-domain nanomagnet with magnetization "M" and easy axis in the y−direction. The nanomagnet is located at distance "a" from the center of the junction along the x-axis, as shown in figure 1.

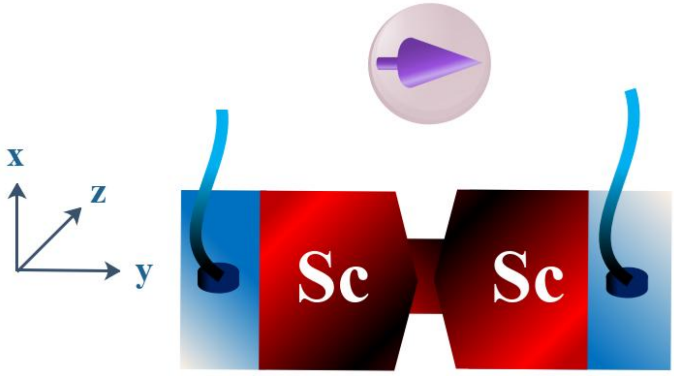

Figure 1 - Schematic diagram showing the geometry of the nanomagnet coupled to Josephson junction.

The magnetic field of the nanomagnet alters the Josephson current, while the magnetic field generated by the Josephson junction acts on the magnetization of the nanomagnet. Thus, there is an electromagnetic interaction between the Josephson junction and nanomagnet. To investigate the dynamics in JJ-coupled nanomagnet, we solve numerically the LLG equation taking into account the effective field due to total tunneling current through the JJ. Thus, the magnetic moment component in the Landau–Lifshitz–Gilbert (LLG) equation is given by

$$\frac{d\boldsymbol{m}}{dt} = -\frac{\omega_{\mathrm{F}}}{(1+\alpha^2)}\left(\boldsymbol{m} \times \boldsymbol{h}_{eff} + \alpha\left[\boldsymbol{m} \times \left(\boldsymbol{m} \times \boldsymbol{h}_{eff}\right)\right]\right) \tag{1}$$

where ω_{F} is ferromagnetic resonance frequency normalized to the characteristic frequency of the Josephson junction $\omega_{\mathrm{c}} = 2\pi I_C R/\Phi_0$, I_c is critical current of the JJ, Φ_0 is the flux quantum, α is the Gilbert damping parameter. The magnetic moment is normalized to the saturation magnetization M_s, and effective field components normalized to magnetic anisotropy energy are given by [1-3]:

$$\begin{aligned} \boldsymbol{h_x} &= 0 \\ \boldsymbol{h_y} &= m_y\,\boldsymbol{\hat{e}_y} \\ \boldsymbol{h_z} &= \epsilon\big(\sin\left(\omega_j t - k m_z\right) + \omega_j + k\dot{m}_z\big)\ \boldsymbol{\hat{e}_z} \end{aligned} \tag{2}$$

where $\epsilon = Gk$, G = $\epsilon_J/K_{an}V_F$ is the Josephson to magnetic energy ratio, $\epsilon_J = \Phi_0 I_c/2\pi$, K_{an} is the magnetic anisotropy constant, V_F is the volume of the nanomagnet, ω_j is the Josephson frequency normalized to ω_c, and $k = \frac{2\pi}{\Phi_0}\mu_0 M_s l\, V_F/\alpha\sqrt{\alpha^2 + l^2}$, this parameter characterize the coupling between the Josephson junction and the nanomagnetic. The first term in $\mathbf{h_z}$ represents the magnetic field, generated by the superconducting current, while the second and third terms represent the magnetic field due to quasiparticle current.

2. Results and discussion

2.1 Study of the reorientation of the easy axis

We investigate the effect of energy ratio parameter "G" on the reorientation of the easy-axis in the superconductor-nanomagnetic system. The initial axis of the magnetic moment is along the y axis (m_y=1).

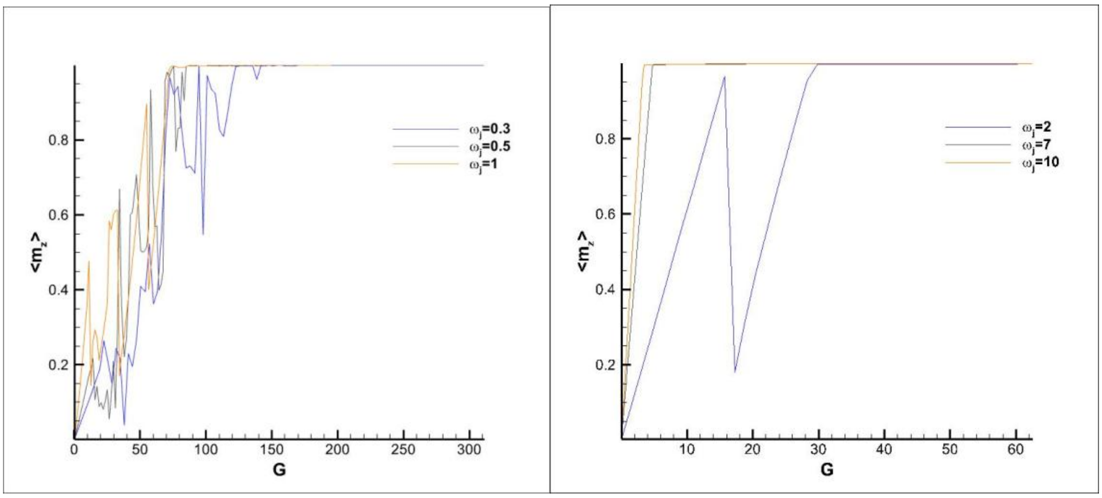

Figure 2 – Dependence of the average value of the magnetic moment along the easy axis on the value of the parameter G at values ω_j = 0.3; 0.5; 1 and ω_j = 2; 7; 10

As can be seen from the figure 2, a complete reorientation of the magnetic moment is achieved by increasing the value of the parameter G. In addition to this, the value of G at which the complete reorientation is reached, takes place at lower value by increasing the Josephson energy. Here, we took ω_F=1. It is worth noting that at ω_j <1, it is observed that the energy of the Josephson junction is insufficient for rapid reorientation of the magnetic moment and reorientation shows irregular behavior. This indicates chaotic feature in this system when ω_j <1 [4]. At ω_j >>1, the reorientation becomes regular and the stable at lower values of G.

It is necessary to investigate the process of reorientation at different values of ω_j and G in different planes of the nanomagnet dynamics. In figures 3,4 and 5 we illustrate a snapshots for the magnetization trajectories at different values of G for ω_j=0.5; 1 and 2 respectively. The trajectory for the magnetic moment in the Josephson junction – nanomagnetic system can be changed dramatically by tunning the energy ratio parameter and Josephson frequency for example it can change from chaotic spirals to a trajectory of sink. The relationship between the parameters is manifested in the fact that an increase in the parameter G leads to a gradual stabilization of the system, moving from chaotic dynamics to more orderly modes of motion of the magnetic moment. The Josephson frequency ω_j plays the role of a regulator of trajectory complexity – as it increases, the system reaches stable states faster. Gilbert damping ensures convergence of solutions to stationary points or stable limit cycles.

Thus, the shape of the magnetization reversal trajectories is the result of the complex influence of all model parameters of the system. Chaotic dynamics is observed mainly at low values of G and intermediate or low values of ω_j, while high values of G and ω_j contribute to the formation of stable periodic regimes. These dependencies make it possible to purposefully control the dynamics of the system by selecting appropriate parameters, which is important for practical applications in spintronics and quantum computing.

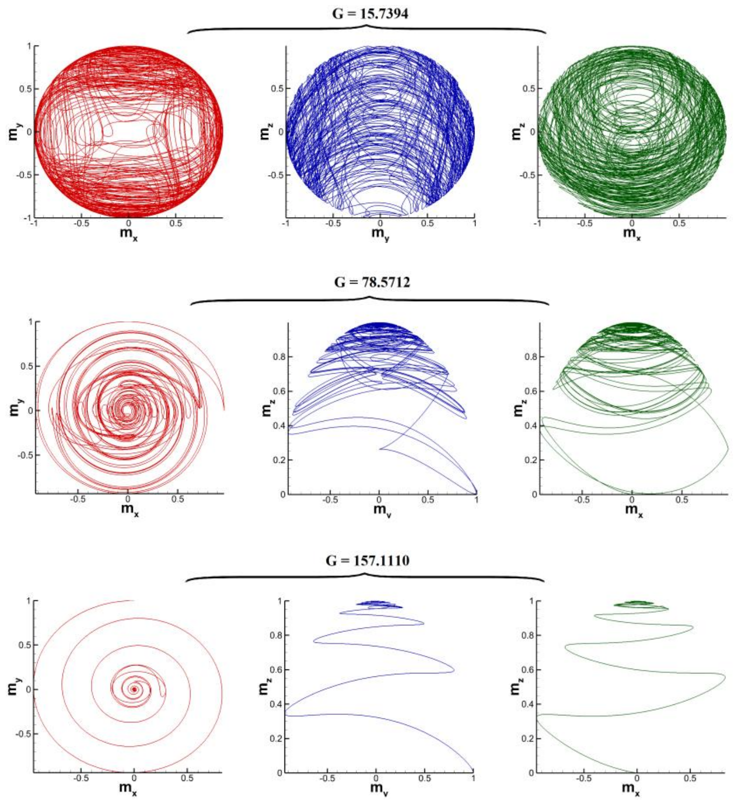

Figure 3 – Demonstrate the 2D trajectory for magnetic moment (m_x, m_y, m_z) at G= 15.7394, G= 78.5712 and G = 157.1110

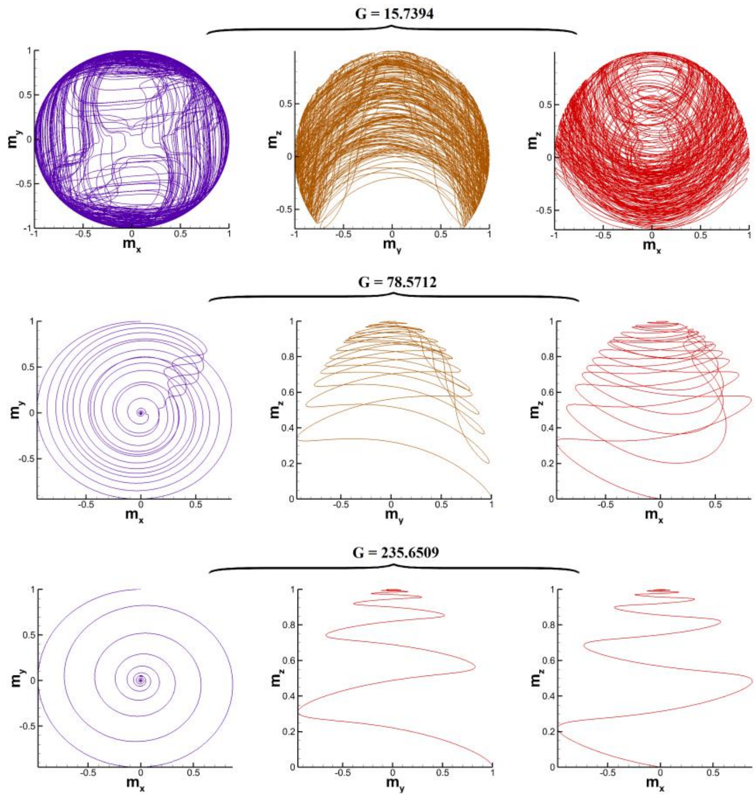

Figure 4 – Demonstrate the 2D trajectory for magnetic moment (m_x, m_y, m_z) at G= 15.7394, G= 78.5712 and G = 235.6509

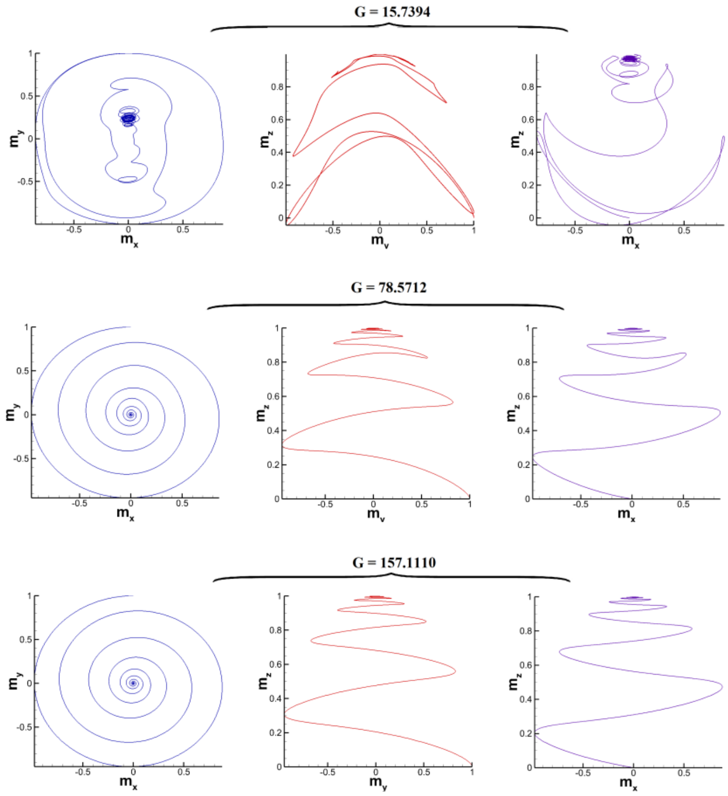

Figure 5 – Demonstrate the 2D trajectory for magnetic moment (m_x, m_y, m_z) at G= 15.7394, G= 78.5712 and G = 157.1110

2.2 Investigation of the influence of initial conditions

The study of the influence of initial conditions on the dynamics of the magnetic moment in the Josephson junction–nanomagnetic system is important for understanding the nonlinear dynamics of the system. In this case (Fig.5-8), we analyze how different initial conditions affect the trajectories of the magnetic moment at a fixed value $\omega_j = \omega_F = 1$ and various values of the G parameter. The study is carried out using the numerical solution of the Landau–Lifshitz–Gilbert (LLG) equations.

As can be seen from the presented figures, the initial conditions significantly affect the dynamics of the magnetic moment only at small values of the parameter G. However, as G increases, the system becomes less sensitive to initial conditions and tends to stable states. This effect is associated with an increased role of the Josephson energy, which prevails over the magnetic energy at high values of G. Increasing the Gilbert parameter (α) affects the dynamics of the Josephson junction–nanomagnetic system, by accelerating the achievement of stable states (see figure 9).

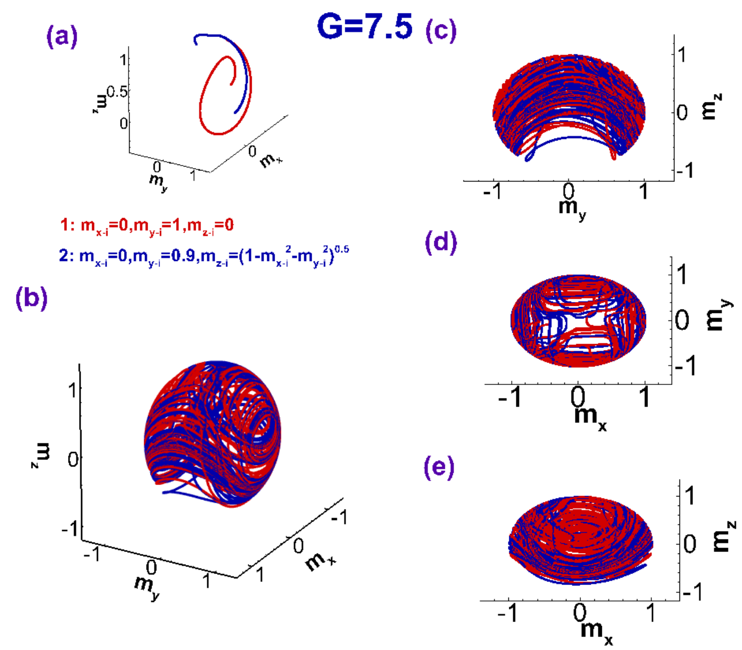

Figure 6 – (a) shows the 3D trajectory of the magnetic moment at the beginning of instance of time. (b) same as in (a) but at a later time. (c,d,f) are the trajectories in various projection. Here, G=7.5, α=0.1.

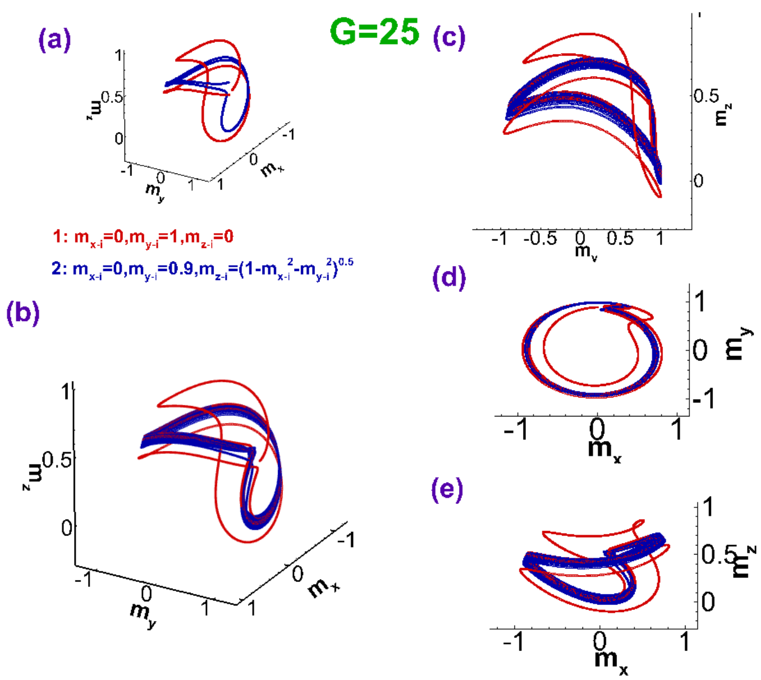

Figure 7 – (a) shows the 3D trajectory of the magnetic moment at the beginning of instance of time. (b) same as in (a) but at a later time. (c,d,f) are the trajectories in various projection. Here, G=25, α=0.1.

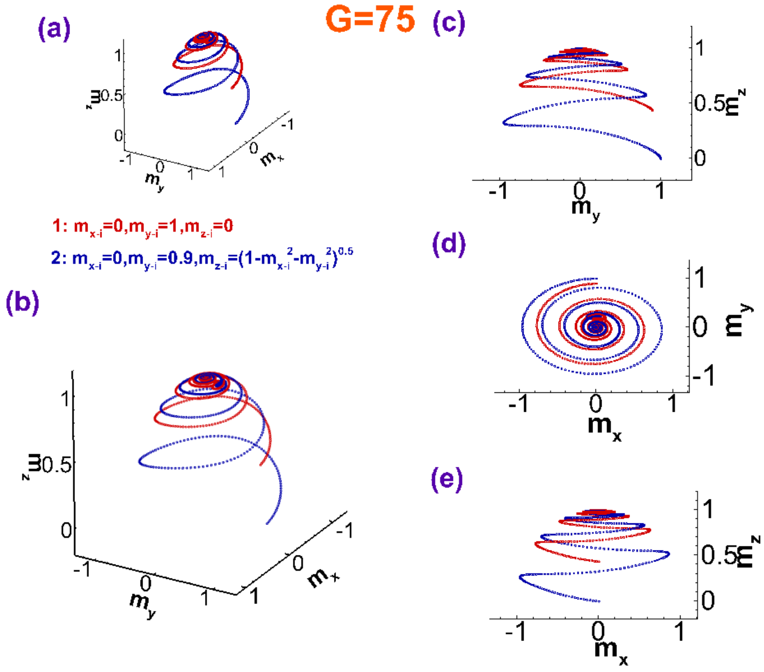

Figure 8 – (a) shows the 3D trajectory of the magnetic moment at the beginning of instance of time. (b) same as in (a) but at a later time. (c,d,f) are the trajectories in various projection. Here, G=75, α=0.1.

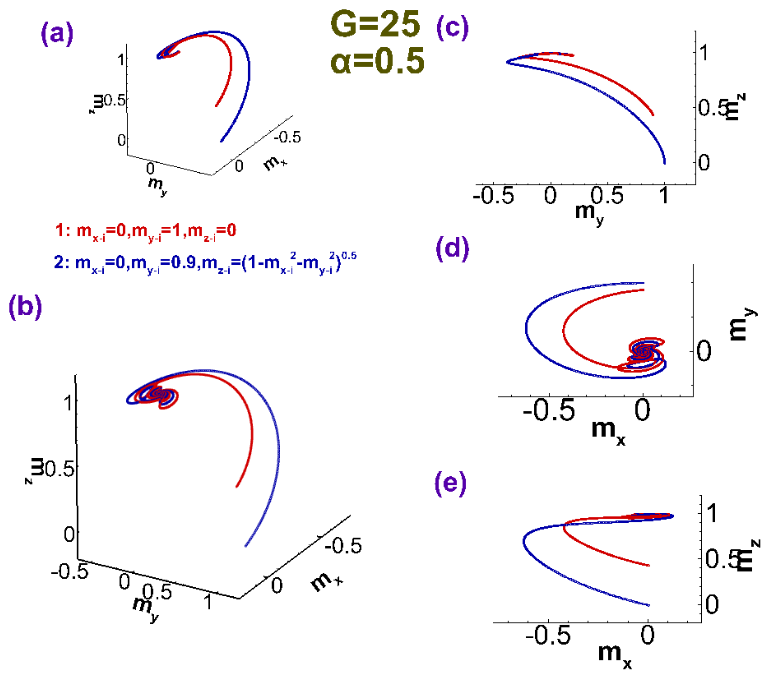

Figure 9 – (a) shows the 3D trajectory of the magnetic moment at the beginning of instance of time. (b) same as in (a) but at a later time. (c,d,f) are the trajectories in various projection. Here, G=7.5, α=0.5.

Thus, the choice of initial conditions is important especially at low values of G, where the system exhibits rich nonlinear dynamics.

2.3 Investigation of stable points

Stable and unstable points play a key role in the dynamics of the Josephson junction – nanomagnetic system. Stable points are the states of a system in which it can stay indefinitely in the absence of external disturbances. For example, the standard stable point corresponds to the lower position of the pendulum, in which the system tends to return to this position after small deviations.

On the contrary, unstable points are characterized by the fact that even a slight deviation from them leads to a system moving away from this state. In a mechanical analogy, this corresponds to the top point of a pendulum, where any small disturbance causes a significant deviation. Mathematically, stability is determined through the projection of velocity towards a stationary point: if this projection is positive, the system is moving towards the point (stable), if it is negative, it is moving away from it (unstable).

An important difference is the behavior of the system under an external periodic disturbance. Under certain conditions, an unstable fixed point can become dynamically stable thanks to the Kapitza method, which divides motion into "fast" and "slow" variables and introduces an effective potential. This explains how the system can stabilize in positions that were initially unstable.

The difference between stable and unstable points is also manifested in their effect on the spontaneous reorientation of the easy-axis magnetization. The transition between different stable states occurs through overcoming energy barriers associated with unstable points, which demonstrates the complex dynamics of the system when changing parameters. After usinging spherical coordinate, by representing (m_x, m_y, m_z) by (Sin(θ) Cos(φ), Cos(θ), Sin(θ)Sin(φ)) and applying Kpaitza method [5,6]. We can find the stable point in the proposed system. As shown in figures 10 when both superconducting current and quasiparticle current are taken into account for the effective field. The red stable points reduced to 1 stable point by increasing the Josephson frequency. This point corresponds to (m_z=1). However, if one neglect the effective field of the quasiparticle current (by neglecting the last two terms in $\mathbf{h_z}$, see Eq(2)), the reverse occurs, i.e, the reorientation is achieved by decreasing the Josephson frequency.

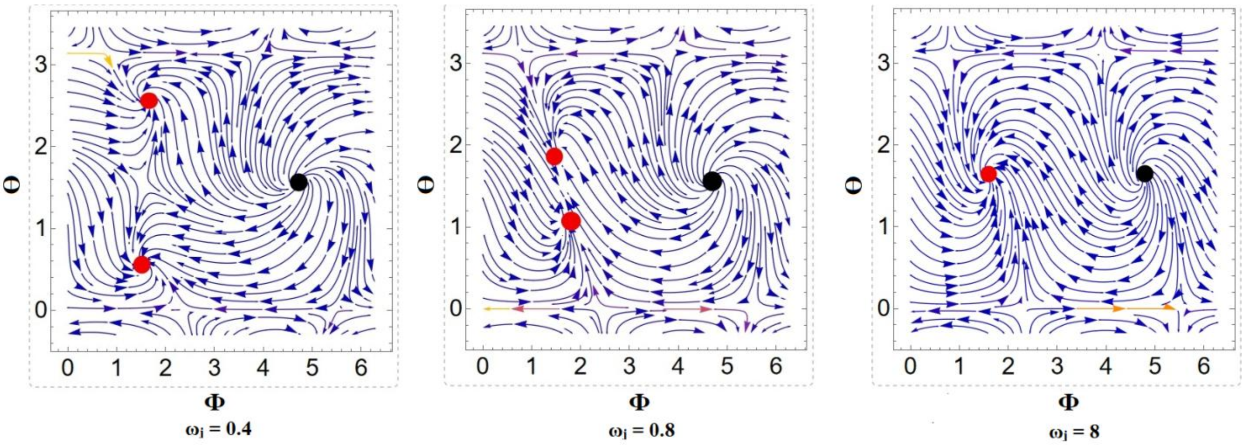

Figure 10 – Phase planes: G=25, determined at the value of ω_j = 0,4; 0,8; 8.

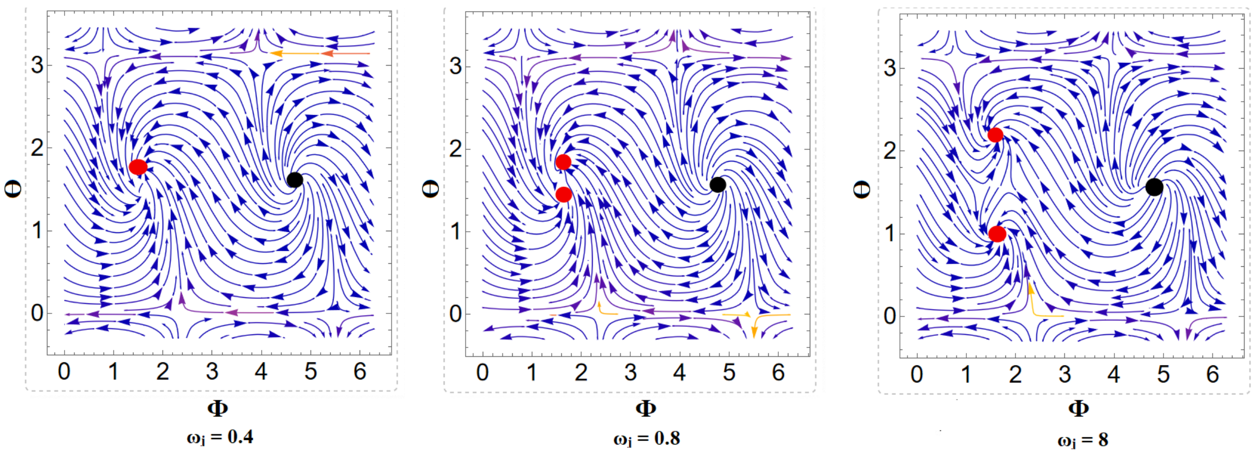

Figure 11 – Phase planes: G=25, determined at the value of ω_j=0,4; 0,8; 8.

The behavior of the system is explained by the influence of the effective field caused by the total current, which includes both the superconducting current and the quasiparticle current. The effective field consists of two main components: the first is related to the overcurrent, and the second and third components depend on the quasiparticle current. These latter components are proportional to ω_j, which increases their effect with increasing Josephson frequency. This effect contributes to the dynamic stabilization of the system and ensures the achievement of a new stable point.

Conclusion

We investigate the dynamic properties of a system consisting of a Josephson junction and a nanomagnetic, with an emphasis on analyzing the behavior of the magnetic moment depending on model parameters such as the energy ratio G and the Josephson frequency ω_j.

Special attention is paid to the stability of the system, the influence of initial conditions, and the role of external factors such as the quasiparticle current. The study is aimed at understanding the mechanisms of reorientation of the easy axis of the magnetization and the conditions under which the system demonstrates controlled dynamics. In addition to this, the analysis shows that if both the superconducting current and the quasiparticle current are taken into account, an increase in the frequency and amplitude of the alternating current leads to a complete reorientation of the easy-axis magnetization, similar to the Kapitsa pendulum. In the absence of the contribution of quasiparticles, the system exhibits behavior similar to the Kapitsa function, where in order to reach a new stable point, it is necessary to increase the parameter G and decrease ω_j. Thus, taking into account all the components of the effective field is key to managing the dynamics of such system.

Understanding the processes that control the dynamics of the magnetic moment opens up new possibilities for creating devices capable of operating at high switching speeds and low power consumption. In particular, the study of stable and unstable points makes it possible to develop methods for controlling the orientation of the magnetic moment, which is a key aspect for the functioning of spin valves and other devices.

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References

1. L. Cai and E. M. Chudnovsky, Interaction of a nanomagnet with a weak superconducting link, Phys. Rev. B 82, 104429 (2010).
2. R. Ghosh, M. Maiti, Yu. M. Shukrinov, and K. Sengupta, Magnetization-induced dynamics of a Josephson junction coupled to a nanomagnet, Phys. Rev. B 96, 174517 (2017).
3. Yu. M. Shukrinov, M. Nashaat, I. R. Rahmonov, and K. V. Kulikov, Ferromagnetic resonance and the dynamics of the magnetic moment in a “Josephson junction nanomagnet” system, JETP Lett. 110, 160 (2019).
4. M. Nashaat, M. Sameh, A. E. Botha, K. V. Kulikov, and Yu. M. Shukrinov, Bifurcation structure and chaos in dynamics of nanomagnet coupled to Josephson junction, Chaos 32, 093142 (2022).
5. K. V. Kulikov, D. V. Anghel, A. T. Preda, M. Nashaat, M. Sameh, and Yu. M. Shukrinov, Kapitza pendulum effects in a Josephson junction coupled to a nanomagnet under external periodic drive, Phys. Rev. B 105, 094421 (2022).
6. K. V. Kulikov, D. V. Anghel, M. Nashaat, M. Dolineanu, M. Sameh, and Yu. M. Shukrinov, Resonance phenomena in a nanomagnet coupled to a Josephson junction under external periodic drive, Phys. Rev. B 109, 014429 (2024).