JOINT INSTITUTE FOR NUCLEAR RESEARCH

Veksler and Baldin laboratory of High Energy Physics

# FINAL REPORT ON THE INTEREST PROGRAMME

*Active role of gluons in hadron interactions (part 2)*

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**Participation period:**
March 02 - April 19

**Dubna 2026**

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# 1 Abstract

Multiplicity distributions in proton-proton interactions are analysed using the Gluon Dominance Model. This model indicates a recombination mechanism of hadronization and a gluon fission. The quark–gluon jets are described as Markov branching processes. The hadronization phase is characterised with a binomial distribution. A fit to Fermilab bubble chamber data obtained at 100 GeV, 300 GeV and 800 GeV energy is carried out. The experimental data from SERP-E-190 is also fitted by GDM.

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# 2 Introduction

Multplicity is the number of secondaries $n$ in the multiparticle production process:

$$p + p \to c_1 + c_2 + ... + c_n.$$

The main feature of this process is that there are already six initial valence quarks before the interaction. It is well known that the mass of the proton is much greater than the sum of the masses of the quarks from which it is composed. Therefore, there are numerous gluons and quark-antiquark pairs (sea quarks) within the proton, which make a significant contribution to the formation of the final hadrons.

The parameters of TSM calculated taking into account all valence quarks and a few gluons in comparison with our previous results obtained for $e^+e^-$ -annihilation differ greatly. For example, the hadronization parameter $n_g^h << 1$ for pp-interactions and $n_g^h \sim 1$ in the case of $e^+e^-$-annihilation. However, the hadronization process should occur the same way in both cases. The parameter of hadronization that agrees with the one obtained for electron-positron annihilation is achieved only by completely excluding valence quarks from consideration.

Thus, valence quarks stay in the leading particles and do not participate in the formation of final hadrons. The multi-particle production is implemented by active gluons (we call gluons that form hadrons active). The model which describes such behaviour is called gluon dominance model (GDM).

Active gluons can give a branch, which is described by Farry distribution. Similarly to the case of electron-positron annihilation, there are three elementary processes which contribute to the quark or gluon distributions into QCD jets:

1. gluon fission: $g \to g + g$;
2. quark bremsstrahlung: $q \to q + g$;

3. quark pair creation: $g \to q + \overline{q}$.

Further we neglect the quark-antiquark pair production, because it gives the smallest part in the process. In addition, the Furry distribution does not take into account quark bremsstrahlung [1].

The hadronization (second stage) is described by the binomial distribution.At energies below 9 GeV the second correlation moment has negative value. We also analysed two schemes: with gluon fission and without it.

# 3 Model with gluon fission

We assume that all secondary particles are born due to gluons. In other words, the quarks from protons do not participate in the creation of the final hadrons. Let $k$ be the number of gluons that divide (active gluons). In the case of elastic scattering, active gluons are absent. Let these gluons be characterised by a Poisson distribution (shows the probability that k gluons can be produced from the original parton):

$$P_k = \frac{e^{-\overline{k}}\overline{k}^k}{k!} \tag{1}$$

The choice of Poisson distribution is based on the analogy with nuclear physics processes (neutron evaporation, radioactive decay). Similarly to the aimed parameter $\lambda$ which determines the probability of releasing a specific number of neutrons from an excited compound nucleus, the $\overline{k}$ determines the probability of releasing a specific number of active gluons from original parton.

The analysis of experimental data from $p^+p^-$ collisions shows that the second correlation moment of charged particles for MD change sign from negative at low energies to positive at higher energies (see Figure 1). For binomial distribution $f_2 > 0$, for Farry distribution $f_2 < 0$ and for Poisson $f_2 = 0$. Thus, according to our model, in the low-energy region the gluon branching process predominates, while in the high-energy region hadronization prevails.

Let us construct the multiparticle distribution for final hadrons in our TSM model. For this, it is necessary to generalize the Furry distribution for the case of a jet developing from a single gluon obtained in [1] for the case of m gluons.

Gluon fission can lead to the formation of gluon jets, which will result in $m$ gluons in the final state. These gluon jets are described by the Furry distribution. The generating function for the development of a jet from a single initial gluon is given by:

$$G(z) = \frac{z}{\overline{m}}\left(1 - z\left(1 - \frac{1}{\overline{m}}\right)\right)^{-1} \tag{2}$$

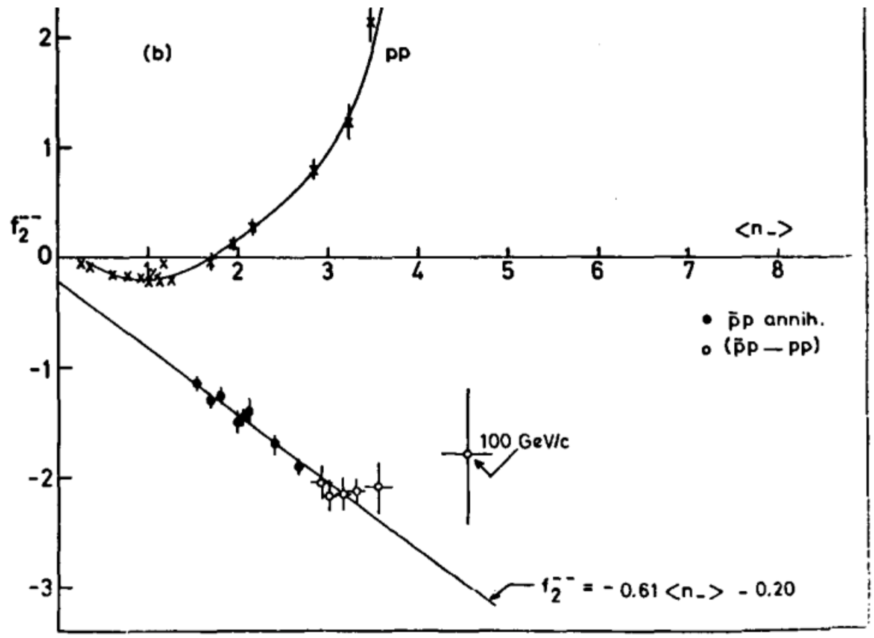


Figure 1: Dependence of the second correlation moment $f_2 = \overline{n(n-1)} - \overline{n}^2$ from mean multiplicity [2]

Considering that the branching (the process of gluon jet development) of gluons occurs independently, in the case of k initial active gluons we obtain:

$$G(z) = \frac{z^k}{\overline{m}^k}\left(1 - z\left(1 - \frac{1}{\overline{m}}\right)\right)^{-k} \tag{3}$$

Let us make use of the connection between the generating function and the distribution (5), as well as Leibniz's formula for the n-th derivative of a product of functions (6):

$$P_m = \frac{1}{m!}\frac{\partial^{(m)}}{\partial z^m}\Big|_{z=0} \tag{4}$$

$$(f \cdot g)^{(n)} = \sum_{k=0}^{n} C_n^k f^{(k)} g^{(n-k)} \tag{5}$$

We obtain a distribution describing the development of a gluon jet in the case of $k$ initial gluons:

$$P_m = \frac{1}{\overline{m}^k}\sum_{p=0}^{m} C_k^{m-p} C_{k+p-1}^{p}\left(1 - \frac{1}{\overline{m}}\right)^p \tag{6}$$

Or in explicit form

$$P_m = \frac{1}{\overline{m}^k}\frac{k!}{(k-1)!}\sum_{p=0}^{m}\frac{(k+p-1)!}{p!(m-p)!(k-(m-p)!))}\left(1 - \frac{1}{\overline{m}}\right)^p \tag{7}$$

Knowing that the factorial may only involve a number greater than or equal to zero, let us estimate the value of the summation index p:

$$m-k \leq p \leq m, \quad k+p \geq 1 \tag{8}$$

From this we obtain that the only possibility to get a distribution describing physical solutions arises when $p = m - k$.

Thus, the Furry distribution for $k$ initial gluons has the following form:

$$P_m = \frac{1}{\overline{m}^k}\frac{(m-1)!}{(m-k)!(k-1)!}\left(1-\frac{1}{\overline{m}}\right)^{m-k} = \frac{C_{m-1}^{k-1}}{\overline{m}^k}\left(1-\frac{1}{\overline{m}}\right)^{m-k} \tag{9}$$

In the case of $k = 1$, the resulting expression is the same as the distribution function given in [1], which describes the gluon jet.

The process of hadronization will be described with the help of binomial distribution. In this case the hadronic multiparticle distribution will have the following form:

$$P_n^H(z) = C_{N_p}^n\left(\frac{\overline{n}_p^h}{N_p}\right)^n\left(1-\frac{\overline{n}_p^h}{N_p}\right)^{N_p-n} \tag{10}$$

where $C_N^n = \frac{N!}{n!(N-n)!}$ - binomial coefficients, $p = q, g$ - type of parton, $\overline{n}_p^h$- the average hadron yield per parton, $N_p$- maximum secondaries of hadrons are formed from parton on the stage of hadronization.

The corresponding generation function:

$$Q_p^H = \left[1+\frac{\overline{n}_p^h}{N_p}(z-1)\right]^{N_p} \tag{11}$$

In TSM we use hypothesis of soft colourless for quarks and gluons on the second stage and add stage of hadronization to branch stage by means of factorization:

$$P_n(s) = \sum_{m=0} P_m^P(s)P_n^H(m,s), \tag{12}$$

where $P_n(s)$ - resulting MD of hadrons, $P_m^P$ - MD of partons (quarks and gluons), $P_n^H(m,s)$ - MD of hadrons (second stage) from $m$ partons. Generation function (GF) for MD in hadron interactions are determined by convolution of two stages:

$$Q(s,z) = \sum_{m=0} P_m^P(s)(Q^H(z))^m = Q^P(s, Q^H(z)), \tag{13}$$

$$P_n(s) = \frac{1}{n!}\frac{\partial^n}{\partial z^n}(Q^P(s,Q^H(z)))|_{z=0}, \tag{14}$$

where $Q^H$ and $Q^P$ - GF for MD at hadronization stage and in QGS.

Let us also take into account the effect of leading protons by reducing the number of final charged particles n by 2. Moreover, we do not consider gluon fusion, so the lower limit for the sum over the final gluons produced as a result of the branching of active gluons is m=k.

As a result, we will obtain the final distribution describing the entire process, where $P_k$ is defined by formula (1) and $P_m$ by formula (10):

$$P_{n-2}(s) = \sum_{k=1}^{MK} \sum_{m=k}^{MG} P_k P_m C_{\delta mN}^{n-2} \left(\frac{\overline{n}^h}{N}\right)^{(n-2)} \left(1 - \frac{\overline{n}^h}{N}\right)^{\delta mN-(n-2)} \tag{15}$$

$$P_{\tilde{n}}(s) = \sum_{k=1}^{MK} \frac{e^{-\overline{k}}\overline{k}}{k!} \sum_{m=k}^{MG} \frac{C_{m-1}^{k-1}}{\overline{m}^k} \left(1 - \frac{1}{\overline{m}}\right)^{m-k} C_{\delta mN}^{\tilde{n}} \left(\frac{\overline{n}^h}{N}\right)^{\tilde{n}} \left(1 - \frac{\overline{n}^h}{N}\right)^{\delta mN-\tilde{n}}, \tag{16}$$

where $\tilde{n} = n - 2$.

Hadronization occurs only for a part of the gluons, so we enter the $\delta$ parameter – the fraction of gluons that fragment into hadrons. It is approximately equal to 47

# 4 Model without gluon fission

Processes with not very high multiplicity (less than 20 charged particles in final state) are described quite well by a model without gluon fission. In this case:

$$P_{n-2}(s) = \sum_{k=1}^{MK} \frac{e^{-\overline{k}}\overline{k}}{k!} C_{kN}^{n-2} \left(\frac{\overline{n}^h}{N}\right)^{(n-2)} \left(1 - \frac{\overline{n}^h}{N}\right)^{kN-(n-2)} \tag{17}$$

# 5 Fitting

Multiplicity distribution $P_n$ obtained in the experiments is the ratio of cross-sections:

$$P_n = \frac{\sigma_n}{\sigma_{inel}},\ where\ \sigma_{inel} = \sigma_{tot} - \sigma_{el} \tag{18}$$

Inelastic processes are all processes that lead to the destruction of initial particles and the creation of new ones. Inelastic processes are distinguished from elastic ones, and their cross-section is measured using an experimental method.

$$\sigma_{inel} = \sum_{n}^{N} \sigma_n \tag{19}$$

| $E_{beam}$ | $50GeV$ | $100GeV$ [3] | $300GeV$ [4] | $800GeV$ [5] |
|---|---|---|---|---|
| $n_{ch}$ | $(P_n \pm \triangle P_n) \cdot 10^2$ | $(P_n \pm \triangle P_n) \cdot 10^2$ | $(P_n \pm \triangle P_n) \cdot 10^2$ | $(P_n \pm \triangle P_n) \cdot 10^2$ |
| 2 | $19.16 \pm 3.12$ | $15.64 \pm 1.06$ | $7.32 \pm 1.56$ | $4.75 \pm 1.78$ |
| 4 | $30.17 \pm 2.57$ | $27.60 \pm 1.06$ | $15.49 \pm 0.68$ | $11.51 \pm 0.40$ |
| 6 | $25.65 \pm 2.24$ | $26.46 \pm 1.02$ | $17.81 \pm 0.74$ | $13.86 \pm 0.45$ |
| 8 | $16.11 \pm 1.54$ | $21.14 \pm 0.89$ | $17.87 \pm 0.74$ | $14.78 \pm 0.46$ |
| 10 | $5.41 \pm 0.38$ | $14.08 \pm 0.67$ | $15.52 \pm 0.68$ | $14.30 \pm 0.46$ |
| 12 | $2.53 \pm 0.18$ | $6.38 \pm 0.43$ | $11.19 \pm 0.58$ | $13.06 \pm 0.44$ |
| 14 | $0.75 \pm 0.06$ | $3.06 \pm 0.31$ | $7.75 \pm 0.49$ | $10.03 \pm 0.36$ |
| 16 | $0.17 \pm 0.02$ | $0.79 \pm 0.15$ | $3.45 \pm 0.37$ | $6.82 \pm 0.25$ |
| 18 | $0.03 \pm 0.005$ | $0.26 \pm 0.08$ | $2.01 \pm 0.22$ | $4.45 \pm 0.23$ |
| 20 | $0.005 \pm 0.002$ | $0.04 \pm 0.04$ | $0.79 \pm 0.15$ | $2.91 \pm 0.19$ |
| 22 | $0.001 \pm 0.0008$ | - | $0.55 \pm 0.12$ | $1.57 \pm 0.12$ |
| 24 | $0.0002 \pm 0.0003$ | - | $0.13 \pm 0.08$ | $0.89 \pm 0.09$ |
| 26 | - | - | $0.89 \pm 0.09$ | $0.39 \pm 0.06$ |
| 28 | - | - | $0.01 \pm 0.01$ | $0.24 \pm 0.06$ |
| 30 | - | - | - | $0.09 \pm 0.03$ |
| 32 | - | - | - | $0.06 \pm 0.03$ |

Table 1: Experimental data

Multiplicity distribution $P_n$ for model without gluon fission has the folowing form:

$$P_n = \Omega \sum_m \frac{e^{-\overline{m}} \overline{m}^m}{m!} C_{mN_g}^{n-2} \left( \frac{\overline{n_g}^h}{N_g} \right)^{(n-2)} \left( 1 - \frac{\overline{n_g}^h}{N_g} \right)^{mN-(n-2)} \quad (20)$$

We used model without gluon fission (20) to fit experimental data obtained at Fermilab bubble chamber for 100 GeV, 300 GeV and 800 GeV energy [3, 4, 5]. For our fitting we used Fumili2 minimization CERN ROOT package [6].

Experimental data are presented in the form of topological cross-sections for the process of the production of n charged particles. In order to proceed to the multiplicity distributions and reassess the errors, it is necessary to use the following formulas:

$$P_n = \frac{\sigma_n}{\sigma_{tot}}, \quad \triangle P_n = \sqrt{\left( \frac{\triangle \sigma_n}{\sigma_{tot}} \right)^2 + \left( \frac{\triangle \sigma_{tot} \cdot \sigma_n}{\sigma_{tot}^2} \right)^2} \quad (21)$$

$$\frac{\triangle P_n}{P_n} = \sqrt{\left( \frac{\triangle \sigma_n}{\sigma_n} \right)^2 + \left( \frac{\triangle \sigma_{tot}}{\sigma_{tot}} \right)^2} \quad (22)$$

The result of the recalculation of the experimental data is presented in the table 1.

| $E_{beam}, GeV$ | MG | $\overline{m}$ | $\overline{n}^h$ | $N$ | $\Omega$ | $\chi^2$ | Ndf |
|---|---|---|---|---|---|---|---|
| 50 | 7 | 2.42 | 1.56 | 2.25 | 1.64 | 14.14 | 8 |
| 100 | 7 | 2.56 | 1.83 | 3.04 | 2.24 | 4.22 | 5 |
| 300 | 11 | 2.88 | 2.26 | 4.42 | 1.99 | 14.02 | 9 |
| 800 | 15 | 3.23 | 2.53 | 13.98 | 2.01 | 27.84 | 11 |

Table 2: Parameters of TSM

Table 2 shows the MG (maximal number of possible gluons created on the first stage) we used during our analysis. The results of comparison of model expression (21) with experimental data are represented in table 2 (parameters of TSM, chi squares, number of degrees of freedom) and on figures 2-5.

Table 1 shows that the average multiplicity of gluons $\overline{m}$ has tendency to insignificantly rise with energy. Maximum number of hadrons formed from one parton on the stage of hadronization $N$ also growth with energy.

The value $\overline{n}_g^h$ in contrast to case of $e^+e^-$ annihilation becomes bigger than 1, that means that the partons transform to hadrons by the recombination mechanism. The recombination is specific for the hadron interactions. Thus, the results obtained in our model agree with those obtained in other theories.

The parameter $\Omega$ must equal 2 to satisfy charge conservation. Since we use leading particles approximation we suppose that all hadrons born as a result of proton collisions must collectively contribute an electrically neutral component. This necessitates that all charged particles are produced in pairs.

The relatively large chi-square values can be partly explained by the small statistics of the experimental data obtained using bubble chambers.

We also used model with gluon fission (23) to fit experimental data obtained for 50 GeV. The region of low multiplicity for this energy was measured by the Mirabelle Collaboration, the high multiplicity region was supplemented by the SVD Collaboration[7]. The results of fitting are represented in table 2.

$$P_n(s) = \Omega_1 \sum_{m=0}^{MG_1} \frac{e^{-\overline{m}}\overline{m}^m}{m!} C_{mN_g}^{n-2} \left(\frac{\overline{n_g}^h}{N_g}\right)^{(n-2)} \left(1 - \frac{\overline{n_g}^h}{N_g}\right)^{mN-(n-2)} + \Omega_2 \sum_{m=0}^{MG_2} \frac{e^{-\overline{m}}\overline{m}^m}{m!} C_{2mN_g}^{n-2} \left(\frac{\overline{n_g}^h}{N_g}\right)^{(n-2)} \left(1 - \frac{\overline{n_g}^h}{N_g}\right)^{2mN-(n-2)} \quad (23)$$

Given that from the normalization condition $\Omega_1+\Omega_2 = 2$, so $\Omega_2$ can be expressed in terms of $\Omega_1$. In Table 2, $\Omega$ refers to $\Omega_1$.

In the model without gluon fission, at an energy of 50 GeV for high multiplicity,

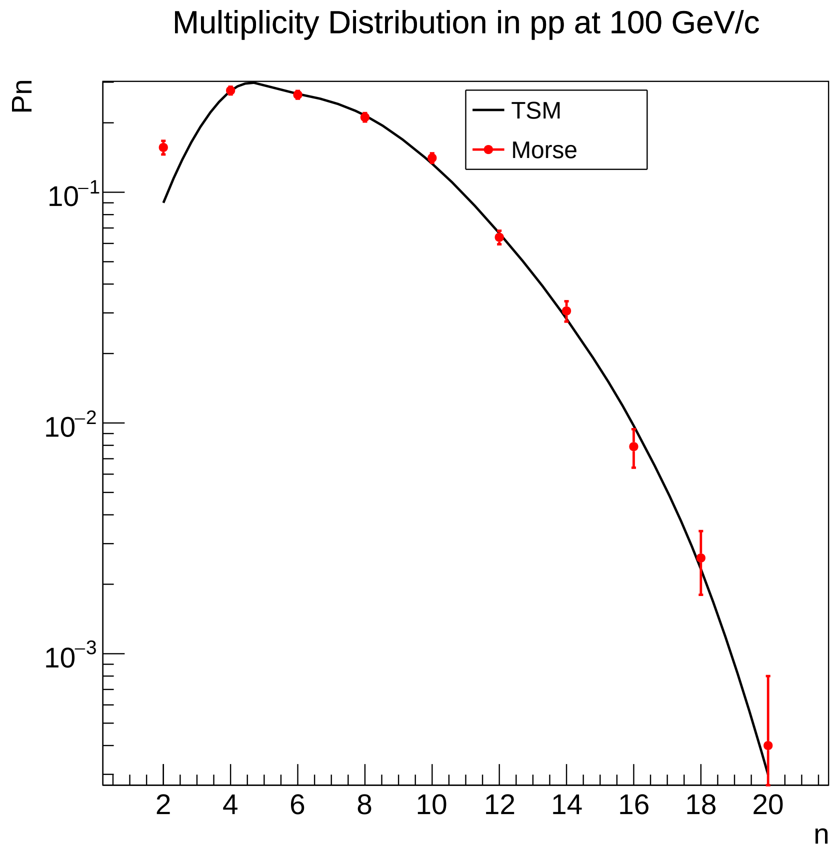


Figure 2: Multiplicity distribution for charged particles in *pp*-collisions for 100 GeV.

a situation arises where $Nm < n - 2$, so experimental values for high multiplicity (20 and more) are not described (see fig.5). At the same time, the model with fission describes these points well, which means that in the process of producing 20 or more final charged particles, the gluon fission plays the main role.

# 6 Charge-exchange in pp interactions

The experiments show that the charge-exchange can be realized at the scattering of protons off hydrogen or nucleus targets. In this case, one of the protons gives its charge to a neutral pion and turns into a neutron:

$$p + p \to p + \pi^+ + n + N_\pi,$$

where $N_\pi$ is the number of secondary pions.

In order to evaluate the charge-exchange, we will consider inelastic processes. Inelastic processes are all processes that lead to the destruction of initial particles and the creation of new ones. The inelastic cross-section can be expressed through

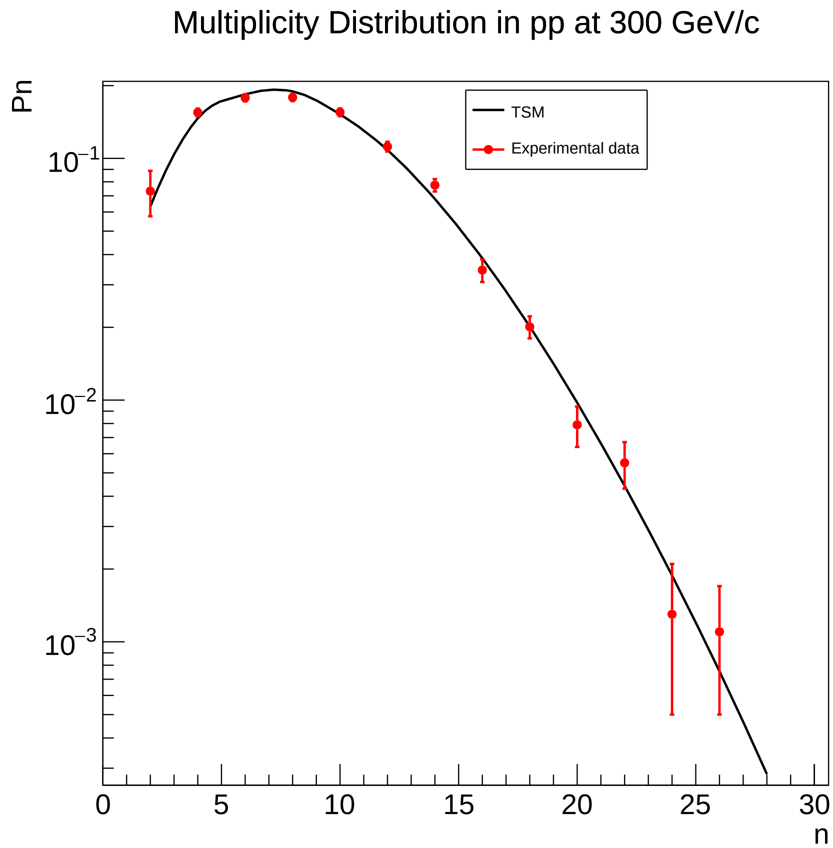


Figure 3: Multiplicity distribution for charged particles in *pp*-collisions for 300 GeV.

elastic cross section. If we consider (17) and suppose $k = 0$ (no active gluons are produced from original parton) we obtain:

$$P_n = e^{-\overline{k}}\overline{k}\left(\frac{\overline{n}^h}{N - \overline{n}^h}\right)^{(n-2)} \tag{24}$$

In the case of elastic reaction, there are no hadrons in final state except initial protons, so

$$P_n = e^{-\overline{k}}\overline{k} \tag{25}$$

That means that $\sigma_{el} \sim e^{-\overline{k}}\overline{k}$. In accordance with the Mirabelle data, the elastic and inelastic cross sections are equal to $\sigma_{2,el} = 6.90$ mb and $\sigma_{2,inel} = 85.71$ mb, respectively. Their ratio is equal to $r = \frac{\sigma_{2,el}}{\sigma_{2,inel}} = 0.82$ and that is why inelastic cross-section can be expressed as $\sigma_{2,inel} = \frac{e^{-\overline{k}}\overline{k}}{r} \cdot$[8].

From the other side:

$$\sigma_{2,inel} = \sigma_{2,inel}^{ch+} + \sigma_{2,inel}^{ch-}, \tag{26}$$

where $\sigma_{2,inel}^{ch+}$ is an inelastic cross-section with charge-exchange and $\sigma_{2,inel}^{ch-}$ – without a charge-exchange.

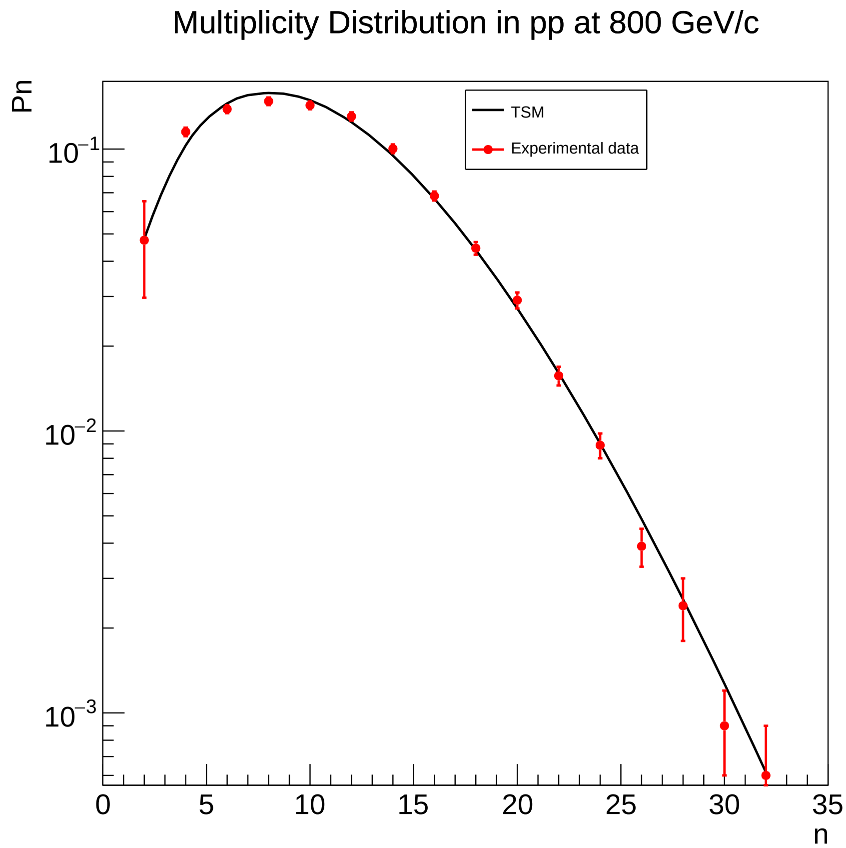


Figure 4: Multiplicity distribution for charged particles in $pp$-collisions for 800 GeV.

Finally coefficient of charge-exchange can be expressed as:

$$q = \frac{\sigma_{2,inel}^{ch+}}{\sigma_{2,inel}} \cdot 100\% \tag{27}$$

# 7 Proton-antiproton annihilation

GDM also describes proton-antiproton annihilation. At low energies the second correlation moment $f_2$ has negative sign in pp- and $p\overline{p}$ interactions. With the growth of energy, $f_2$ changes its sign in the region $\sim$5 GeV for pp and $\sim$ 30 GeV for $p\overline{p}$. The negative sign of $f_2$ evidences the dominance of hadronization under the quark-gluon cascade.

In the experiment for $p\overline{p}$-annihilation formation of three hadron jets is observed: $p\overline{p} \to 3\pi$. These three leading pions can be all neutral (two possibilities, see the scheme below) or there may be two charged and one neutral pion (four possibilities). Also, for initial valence quarks it's possible to get into pairs with quarks that randomly appear and disappear in gluon medium (the so-called superposition state).

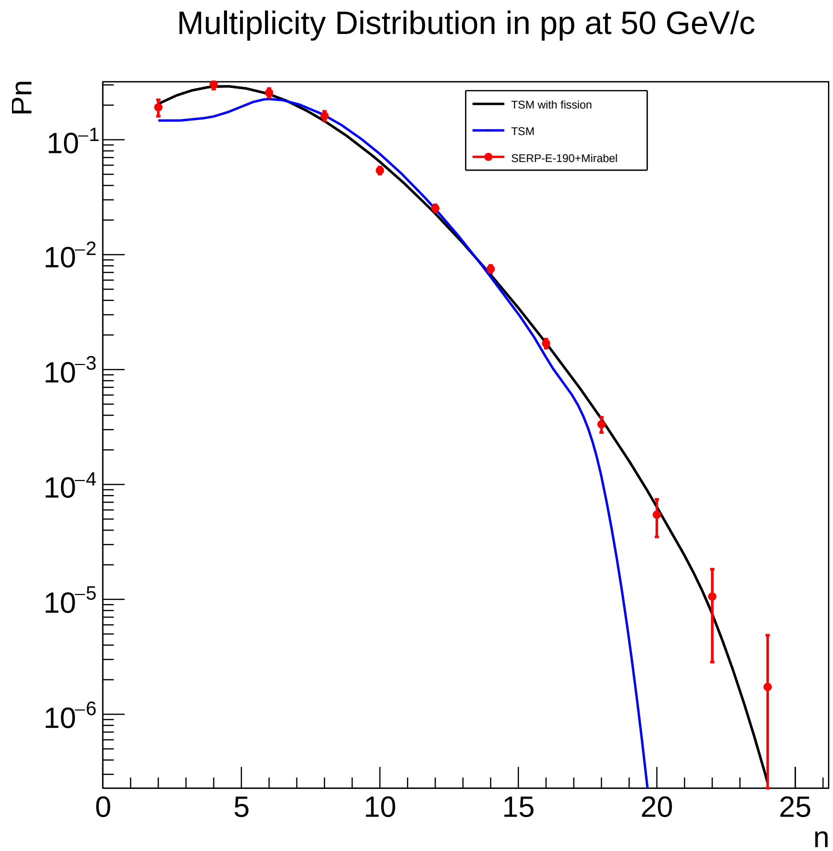


Figure 5: Multiplicity distribution for charged particles in *pp*-collisions for 50 GeV.

We can call these three possibilities for intermediate quark charged topologies as "0", "2" and "4", correspondently.

$p(u_1 u_2 d) + \overline{p}(\overline{u}_1 \overline{u}_2 \overline{d}) \rightarrow \pi_0(u_1 \overline{u}_1) + \pi_0(u_2 \overline{u}_2) + \pi_0(d\overline{d})$
$p(u_1 u_2 d) + \overline{p}(\overline{u}_1 \overline{u}_2 \overline{d}) \rightarrow \pi_0(u_2 \overline{u}_1) + \pi_0(u_1 \overline{u}_2) + \pi_0(d\overline{d})$
$p(u_1 u_2 d) + \overline{p}(\overline{u}_1 \overline{u}_2 \overline{d}) \rightarrow \pi^+(u_1 \overline{d}) + \pi_0(u_2 \overline{u}_1) + \pi^-(d\overline{u}_2)$
$p(u_1 u_2 d) + \overline{p}(\overline{u}_1 \overline{u}_2 \overline{d}) \rightarrow \pi_0(u_1 \overline{u}_1) + \pi^+(u_2 \overline{d}) + \pi^-(d\overline{u}_2)$
$p(u_1 u_2 d) + \overline{p}(\overline{u}_1 \overline{u}_2 \overline{d}) \rightarrow \pi_0(u_1 \overline{u}_2) + \pi^+(u_2 \overline{d}) + \pi^-(d\overline{u}_1)$
$p(u_1 u_2 d) + \overline{p}(\overline{u}_1 \overline{u}_2 \overline{d}) \rightarrow \pi^+(u_1 \overline{d}) + \pi_0(u_2 \overline{u}_2) + \pi^-(d\overline{u}_1)$

By analogy with the case of proton-proton collisions, the multiplicity distribution can be written, taking into account the above, as:

$$P_n(s) = C_0 \sum_{m=1}^{MG} \frac{e^{-\overline{m}}\overline{m}}{m!} C_{mN}^{n} \left(\frac{\overline{n}^h}{N}\right)^n \left(1 - \frac{\overline{n}^h}{N}\right)^{mN-n} + \\ C_2 \sum_{m=1}^{MG} \frac{e^{-\overline{m}}\overline{m}}{m!} C_{mN}^{n-2} \left(\frac{\overline{n}^h}{N}\right)^{(n-2)} \left(1 - \frac{\overline{n}^h}{N}\right)^{mN-(n-2)} + \\ C_4 \sum_{m=1}^{MG} \frac{e^{-\overline{m}}\overline{m}}{m!} C_{mN}^{n-4} \left(\frac{\overline{n}^h}{N}\right)^{(n-4)} \left(1 - \frac{\overline{n}^h}{N}\right)^{mN-(n-4)} \quad (28)$$

# 8 Conclusions

In the case of proton-proton collisions in multiple production, only active gluons participate, while valence quarks remain in the leading particles (protons). This model is called the Gluon Dominance Model. The model parameters for 50, 100, 300, 800 GeV beam energies were found. It was shown that GDM describes multiparticle production processes quite well. For processes that produce less than 20 charged particles, a simplified, non-fission-based scheme can be used.

# References


[1] A. Giovannini. Qcd jets as markov branching processes. *Nuclear Physics B*, 161(2):429–448, 1979.

[2] J.G. Rushbrooke and B.R. Webber. High energy antiparticle-particle reaction differences and annihilations. *Physics Reports*, 44(1):1–92, 1978.

[3] W. M. Morse, V. E. Barnes, D. D. Carmony, R. S. Christian, A. F. Garfinkel, L. K. Rangan, A. R. Erwin, E. H. Harvey, R. J. Loveless, and M. A. Thompson. $\pi^+p$, $K^+p$, and pp topological cross sections and inclusive interactions at 100 gev using a hybrid bubble-chamber-spark-chamber system and a tagged beam. *Phys. Rev. D*, 15:66–74, Jan 1977.

[4] A. Firestone et al. *pp* Interactions at 300-GeV/c: Measurement of the Charged Multiplicity, Total and Elastic Cross-Sections. *Phys. Rev. D*, 10:2080, 1974.

[5] R. Ammar, T. Aziz, S. Banerjee, J.F. Baland, S. Ball, R.C. Ball, C. Bromberg, R. Brun, G.E. Canough, T. Coffin, V. Commichau, R. Davis, T.O. Dershem, R.L. Dixon, H.C. Fenker, S.N. Ganguli, U. Gensch, N. Giokaris, P. Girtler, A.T.

Goshaw, J. Gress, A. Gurtu, V.P. Henri, J.J. Hernandez, J. Hrubec, M. Iori, L.W. Jones, D. Knauss, D. Kuhn, N. Kwak, I.D. Leedom, P. Legros, J. Lemonne, H. Leutz, X. Liu, P.K. Malhotra, J.M. Marraffino, G.E. Mendez, S. Mikocki, R. Miller, T. Naumann, G. Neuhofer, A. Nguyen, M. Nikolic, H. Nowak, P. Pilette, A. Poppleton, J. Poirier, R. Raghavan, K. Rasner, S. Reucroft, W.J. Robertson, B.P. Roe, C.E. Roos, A. Roth, M. Senko, W. Struczinski, A. Subramanian, M.C. Touboul, B. Vonck, L. Voyvodic, W.D. Walker, J.W. Waters, M.F. Weber, M.S. Webster, J. Wickens, and C.F. Wild. Multiplicity of charged particles in 800 gev p-p interactions. *Physics Letters B*, 178(1):124–128, 1986.

[6] Rene Brun and Fons Rademakers. Root — an object oriented data analysis framework. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 389(1):81–86, 1997. New Computing Techniques in Physics Research V.

[7] Ardashev E.N. et al. Topological cross-sections in pp-interactions at 50 gev, 2011.

[8] E. S. Kokoulina. Multiplicity description by gluon model, 2015.

# Acknowledgments

I would like to express my sincere gratitude to my supervisor, Professor Elena Kokoulina, who guided me throughout this project. Her extensive background and deep knowledge in multiplicity allowed me to get a lot of interesting insights.