INTErnational REmote Student Training Program
Joint Institute for Nuclear Research
Dubna, Russia

Analysis and interactive visualization of neutrino event topologies registered in the OPERA experiment

PROJECT REPORT

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Contents

1 Introduction

In contrast to charged leptons like electrons and muons which are readily detected from continuous track defined by ionization of atoms as they pass through matter, neutrinos are never directly observed since they do not interact electromagnetically. They are only detected from weak interactions.[1] Several predication and theories have been put into consideration to describe neutrinos and their relation to their antiparticle. However, It was stated experimentally that neutrino exhibits a flavor that is associated with the charged lepton produced through the weak interactions.[2] Table 1 shows different flavors of neutrinos and their associated charged lepton. Thus, the electron neutrino ν_e is defined as the neutrino produced in a charged current weak interaction along with the electron. Additionally, the weak charged current interaction of ν_e will produce an electron.

For several years, it was assumed the neutrinos are mass-less particles, for experimental evidence associated with β decay.[3] It was observed that the interactions of neutrino/antineutrino produced with positron/electron would produce an electron/positron pair as illustrated in Fig 1. This led to the idea that electron neutrino has some kind of property related to the electron that is conserved in weak interactions. Additionally, beam neutrino experiments have

1. Peter J. Bussey, *Modern Particle Physics, by Mark Thomson*, vol. 55, 3 (2014), 251–252, ISBN: 9781107034266, https://doi.org/10.1080/00107514.2014.907354.

2. Spandan Mondal, "Physics of Neutrino Oscillation," 2015, arXiv: 1511.06752, http://arxiv.org/abs/1511.06752.

3. Olivia Meredith Bitter, "An Introduction to Neutrino Oscillation : A Quantum Mechanical Perspective," 2021, 1–13.

Table 1: A table showing the flavors of different neutrino with their associated charged lepton

Generation I	Generation II	Generation III
e^-	μ^-	τ^-
ν_e	ν_μ	ν_τ

shown that neutrinos are produced from $\pi^+ \longrightarrow \mu^+ + \nu_\mu$ decays.[4]

Additional evidence was provided by branching ration in $\mu^- \longrightarrow e^- + \gamma$ decay. Fig 2 shows the decay process. During this interaction, there exists no decay process which highly suggested that the interaction vertices of $W_{\mu^- \nu}$ and $W_{e^- \nu}$ are different.

By the 1990s, very little knowledge was known about neutrinos distant from their flavors and that they are extremely light(possibly have no mass).[7] However, several experiments reported anomalies in the interaction rates of atmospheric and solar neutrinos. However, data from the super-Kamiokande detector provided compelling evidence of a phenomenon known as neutrino flavor oscillation over a large distance.[8] The subsequent study of neutrino oscillations has been one of the highlights of particle physics in recent years.

Herein, we investigate the phenomenon of neutrino oscillations from an experimental point of view with some results of the theory. We begin by introducing the concept of oscillation and the associated probability of oscillation of a two-state system. Then, we further discuss the OPERA experiment that studied $\nu_\mu \longrightarrow \nu_\tau$ oscillations.

4. Bussey, *Modern Particle Physics, by Mark Thomson*.

7. S. Bilenky, "Neutrino oscillations: From a historical perspective to the present status," *Nuclear Physics B* 908 (2016): 2–13, ISSN: 05503213, https://doi.org/10.1016/j.nuclphysb.2016.01.025, arXiv: 1602.00170, http://dx.doi.org/10.1016/j.nuclphysb.2016.01.025.

8. D. V. Naumov, "Introduction to neutrino physics," *Proceedings of the International Baikal Summer School on Physics of Elementary Particles and Astrophysics 2010*, 2010, 218–262, ISSN: 0029-0181, https://doi.org/10.1007/978-981-287-715-4_1.

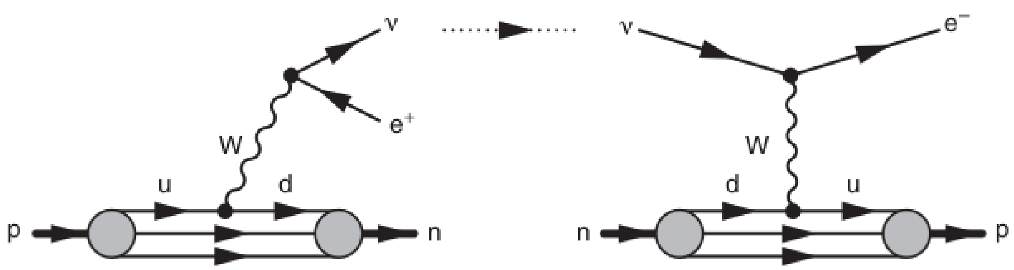


Figure 1: Neutrino production and subsequent detection where the ν_e state is associated with positrons/electrons. retrieved from[5]

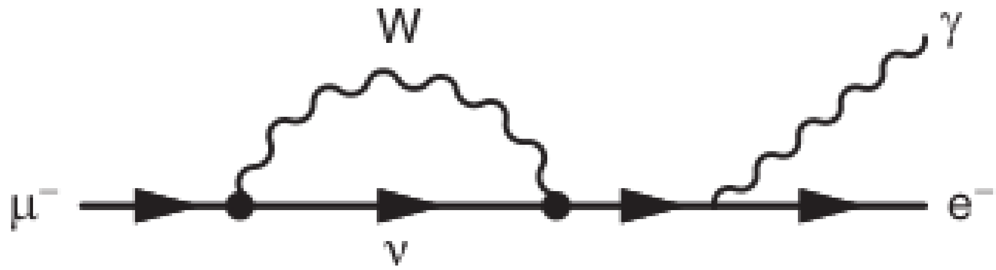


Figure 2: A Feynman diagram of decay of $\mu^- \longrightarrow \gamma + e^-$ where ν_e and ν_μ do not interact. retrieved from.[6]

2 Neutrino Oscillations

The neutrino flavor transformations observed experimentally can be explained by neutrino oscillation which was first introduced by Pontecorvo in 1957.[9] The physical state of the particle (the mass eigenstate) are considered as stationary states of the free particle Hamiltonian[10][11][12].[13]

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi = E\psi$$

where the time evolution of the mass eigenstate has the following form,

$$\psi(\vec{r},t) = \phi(\vec{r})e^{-iEt/\hbar}$$

We will refer to the mass eigenstates as ν_1, ν_2 and ν_3. There is no reason to suspect that the mass eigenstates are those associated with lepton decays produced along with weak interactions. Fig 3 shows the distinction between mass and weak eigenstates. The figure illustrates

9. Bilenky, "Neutrino oscillations: From a historical perspective to the present status."

10. Bilenky.

11. Bitter, "An Introduction to Neutrino Oscillation : A Quantum Mechanical Perspective."

12. G. Fantini et al., "Introduction to the formalism of neutrino oscillations," *The State Of The Art Of Neutrino Physics: A Tutorial For Graduate Students And Young Researchers*, 2018, 37–119, https://doi.org/10.1142/10600, arXiv: arXiv:1802.05781v2.

13. Naumov, "Introduction to neutrino physics."

that upon measuring the mass of the neutrino associated with weak interaction of electron, we will find variations in the measured mass which correspond to the masses eigenstates with some probability. Thus, the system has to be described by a linear combination of the mass eigenstates.

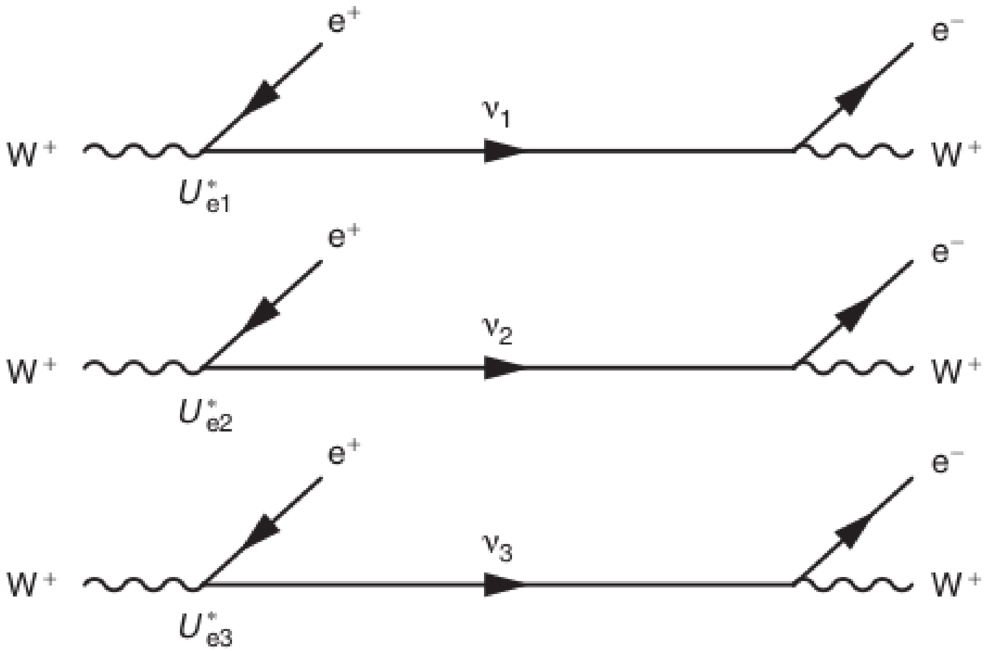


Figure 3: Vertex of $W_{e^+\nu_e}$ as decomposition of mass eigenstates. retrieved from.[14]

Using quantum mechanical theory, we can relate the mass eigenstates with the produced neutrinos through a basis transformation of a unitary matrix as follows,[15]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

15. Bussey, *Modern Particle Physics, by Mark Thomson*.

Hence the electron neutrino state can be described as follows,

$$|\psi_{\nu_e}\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$

Thus, the neutrino state propagates through time until it interacts where the wave function collapses into an eigenstate producing an observable charged lepton of a certain flavor. Since the masses of the eigenstates are not the same phase difference arises through the time of propagation which changes the coefficients of the wave function, thus producing the discovered neutrino oscillation phenomenon. In this manner, a neutrino of a certain flavor can produce charged lepton through weak interactions different from its associated lepton. Fig 4 illustrates this process.

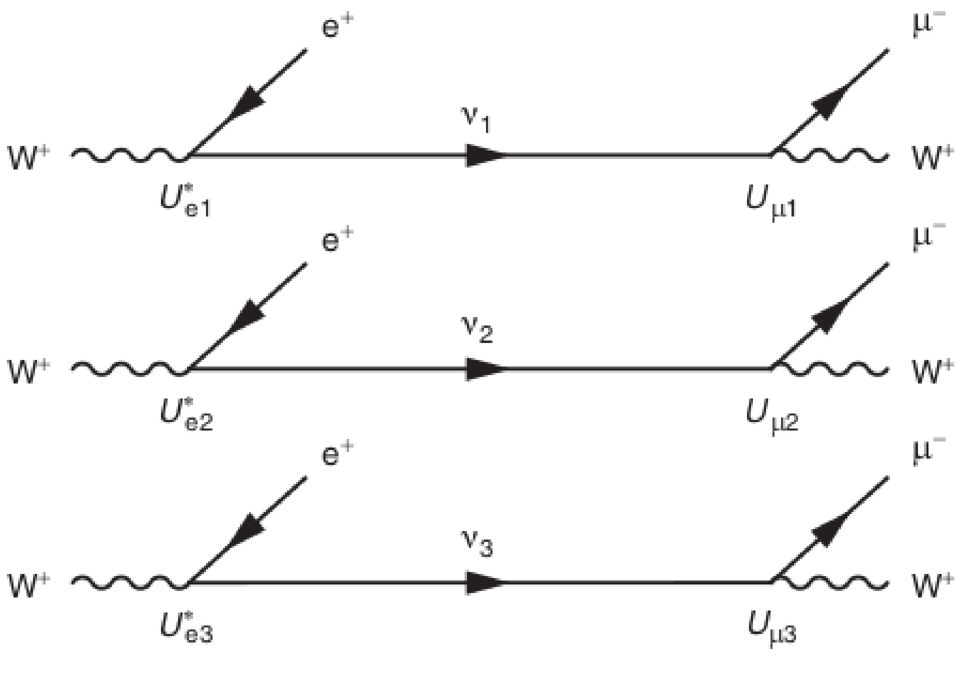


Figure 4: Process of $\nu_e \longrightarrow \nu_\mu$ oscillation. retrieved from.[16]

2.1 Neutrino oscillations of two flavors

Here, we shall consider the oscillation associated with ν_e and ν_μ which are taken to be linear combinations of ν_1 and ν_2,

$$|\nu_1(t)\rangle = |\nu_1\rangle\, e^{i(\vec{p}_1\cdot\vec{r}-E_1 t)}$$

$$|\nu_2(t)\rangle = |\nu_2\rangle\, e^{i(\vec{p}_2\cdot\vec{r}-E_2 t)}$$

Where $(E_1, \vec{p}_1)$ and $(E_2, \vec{p}_2)$ are the associated energy and three momentum of the two states. Since we are considering only a two flavor oscillation we can easily consider a unitary transformation between basis using an arbitrary angle θ. This angle is known as the mixing angle.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Now, suppose that the state wave function of electron neutrino at $t = 0$ can be described through the following combination.

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

Thus, the state will evolve in time as follows,

$$|\psi(\vec{r},t)\rangle = \cos\theta\,|\nu_1\rangle\, e^{i(\vec{p}_1\cdot\vec{r}-E_1 t)} + \sin\theta\,|\nu_2\rangle\, e^{i(\vec{p}_2\cdot\vec{r}-E_2 t)}$$

After time $t = T$ the neutrino would travel a distance of L along x-direction which evolve the state as,

$$|\psi(L,T)\rangle = \cos\theta\,|\nu_1\rangle\, e^{-i\phi_1} + \sin\theta\,|\nu_2\rangle\, e^{-i\phi_2}$$

where

$$\phi_i = E_i T - p_i L$$

However, we can write this in terms of weak eigenstate basis using the the inverse transformation of the above matrix.

$$\begin{aligned} |\psi(L,T)\rangle &= \cos\theta(\cos\theta\,|\nu_e\rangle - \sin\theta\,|\nu_\mu\rangle)e^{-i\phi_1} + \sin\theta(\sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\mu\rangle)e^{-i\phi_2} \\ &= (e^{-i\phi_1}\cos^2\theta + e^{-i\phi_2}\sin^2\theta)\,|\nu_e\rangle - (e^{-\phi_1} + e^{-i\phi_2})\cos\theta\sin\theta\,|\nu_\mu\rangle \\ &= e^{-i\phi_1}\left[(\cos^2\theta + e^{i\Delta\phi}\sin^2\theta)\,|\nu_e\rangle - (1 - e^{i\Delta\phi})\cos\theta\sin\theta\,|\nu_\mu\rangle\right] \end{aligned}$$

where $\Delta\phi = \phi_1 - \phi_2$. From this we can notice that upon evolution of the electron neutrino eigenstate a muon may appear upon interaction due to the presence of $|\nu_\mu\rangle$ term. This will always be true as long as $\Delta\phi \neq 0$. Thus, now we can write the evolved state as,

$$|\phi(L,T)\rangle = c_e |\nu_e\rangle + c_\mu |\nu_\mu\rangle$$

We can now find the probability that an electron neutrino will change into a muon neutrino as,[17][18]

$$\begin{aligned} P(\nu_e \longrightarrow \nu_\mu) &= |\ \langle \nu_\mu|\psi(L,T)\rangle\ |^2 = c_\mu c_\mu^* \\ &= (1 - e^{i\Delta\phi})(1 - e^{-i\Delta\phi}) \cos^2\theta \sin^2\theta \\ &= (2 - e^{i\Delta\phi} - e^{-i\Delta\phi}) \cos^2\theta \sin^2\theta \\ &= \frac{1}{4}(2 - 2\cos(\Delta\phi)) \sin^2(2\theta) \\ &= \sin^2(2\theta) \sin^2\left(\frac{\Delta\phi}{2}\right) \end{aligned}$$

Thus, the probability of oscillation $\nu_e \longrightarrow \nu_\mu$ depends on the mixing angle θ and phase difference $\Delta\phi$. We can further express the probability of oscillation by assuming that the momentum of the two eigenstates is the same $p_1 = p_2$. This permits us to write $\Delta\phi$ as follows,

$$\Delta\phi = (E_1 - E_2)T = p\left[\sqrt{1 + \frac{m_1^2}{p^2}} - \sqrt{1 + \frac{m_2^2}{p^2}}\right]$$

since the mass is very small $m \ll E$. and $T \approx L$ in natural units.

$$\sqrt{1 + \frac{m^2}{p^2}} \approx 1 + \frac{m^2}{2p^2}$$

17. Neutrino Flavour Oscillation et al., "Neutrino Oscillations," 2020, 1–52.
18. Mondal, "Physics of Neutrino Oscillation."

Thus,

$$\Delta\phi \approx \frac{m_1^2 - m_2^2}{2p} L$$

. Thus, now we can write the probability as

$$P(\nu_e \longrightarrow \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{m_1^2 - m_2^2}{4p} L\right)$$

3 OPERA Experiment

The OPERA Experiment is designed to detect the first observation of a ν_τ from a $\nu_\mu \longrightarrow \nu_\tau$ oscillation by implementing a long-baseline beam from CERN to Gran Sassa Laboratory (LNGS), 730 km away. The CNGS beam consists mainly of ν_μ with an energy of 17 GeV. The beam also contain a very small amount of contamination of 2.1% of $\bar{\nu_\mu}$ charged current events and $\sim 0.9\%$ of $\bar{\nu_e}$.[19]

The CNGS neutrino beam is emitted by a 400 GeV/c proton beam from the SPS accelerator.[20] The beam is transported 840 m onto a carbon target That produces kaons and pions. A reflector and a horn system are designed to select positively charged π and K that decay into ν_μ and μ in a long vacuum tube of 1000 m long. Additionally, all hardons are stopped using a hadron stopper after directing the beam to the LNGS lab. An 18 m long block of iron and graphite is designed to permit the passage of only muon neutrinos and muon leptons.

OPERA is designed to identify τ lepton from the topological observation of its decay which is accompanied by a kinematic analysis afterwords. This is performed using real-time detection techniques (ie electronics detectors) with an Emulsion Cloud Chamber (ECC) technique. The ECC detector is made of plates used as targets with an alteration of nuclear emulsion films used as tracking devices.

19. R. Acquafredda et al., "The OPERA experiment in the CERN to Gran Sasso neutrino beam," *Journal of Instrumentation* 4, no. 4 (2009), ISSN: 17480221, https://doi.org/10.1088/1748-0221/4/04/P04018.

20. Acquafredda et al.

The electronic detectors trigger data acquisition by identifying and measuring trajectories of charged particles and locating the brick where the interaction occurred. The interaction brick is then extracted with two interface emulsion films called "Changeable sheets", attached to the downstream face of the brick. If the CS showed tracks of neutrino interactions, the films of the brick will be fully analyzed to locate the interaction vertex. The analysis of the topology at the primary vertex will then select possible τ candidates.Fig 5, retrieved from,[21] shows a ν_τ charged current interaction with a long decay of τ lepton as it appears in CS, OPERA bricks, and scintillator trackers.

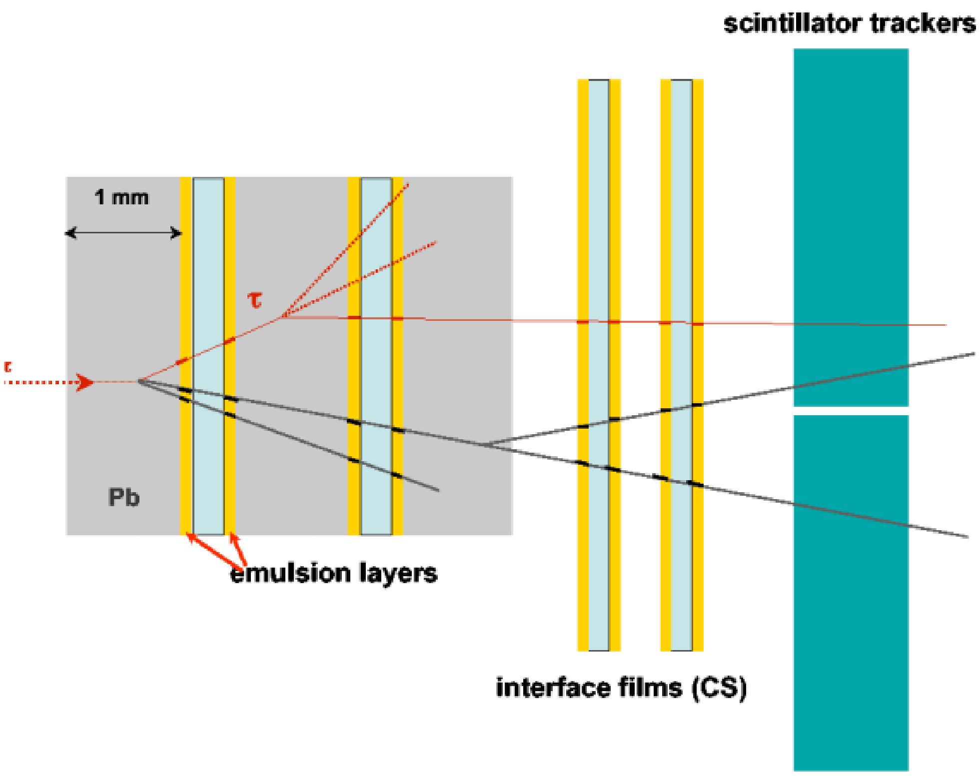


Figure 5: Schematic view of a ν_τ CC event as it would appear in the scintillator trackers, CS, and OPERA bricks.

21. Acquafredda et al., "The OPERA experiment in the CERN to Gran Sasso neutrino beam."

Fig 6, retrieved from,[22]shows a schematic diagram of the structure of the OPERA detector. Each wall contains 2912 bricks supported by a light stainless steel structure followed by scintillators. The instrumented target is followed by a magnetic spectrometer that consists of a large iron magnet and Resistive plate chambers (RPC) where the detection of charged particles inside the magnetized iron is measured by six stations of drift tubes.

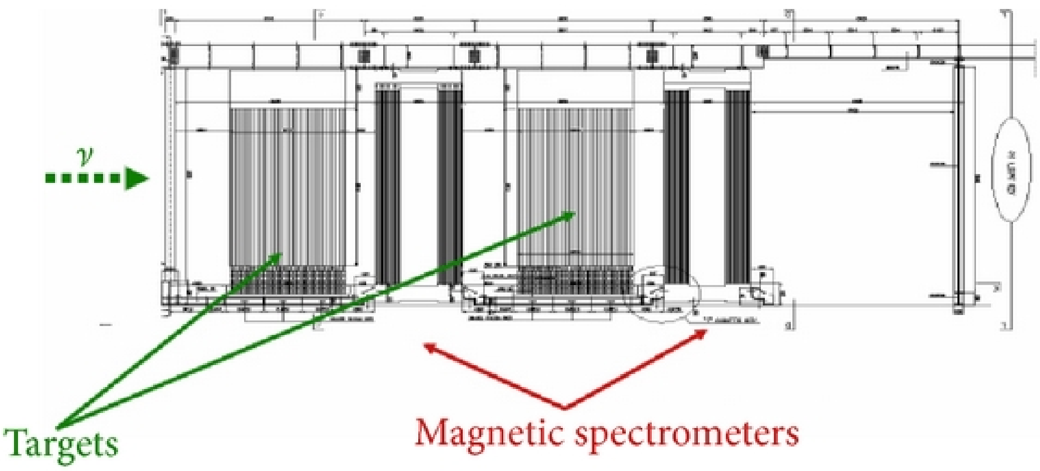


Figure 6: schematic diagram illustrating the structure of the OPERA detectors and its vital components.

The OPERA experiment contains about 150000 ECC bricks with 110,000 m^2 emulsion films and 105,000 m^2 lead plates. The scanning of the events are performed with more than 30 fully automated microscopes located in several different laboratories around the world.

In the following section, we analyze some of the data of the OPERA experiment extracted from CERN Open Data Portal.

22. G. Giacomelli et al., "Neutrino oscillations in the atmospheric parameter region: From the early experiments to the present," *Advances in High Energy Physics* 2013 (2013), ISSN: 16877357, https://doi.org/10.1155/2013/464926.

4 Results and Discussion

In this section, we go through the tasks required in the project by analyzing the outcomes of each task individually. All the data sets have been obtained from CERN open data portal for the OPERA experiment. The data sets for the following three studies are labeled as follows:

- Emulsion data for neutrino-induced charmed hadron production studies
- Emulsion data for multiplicity studies
- Emulsion data for neutrino tau appearance studies

4.1 Analysis of Emulsion Data of ν-induced charmed hadron production

In this study, we assess the validity of the ν_τ appearance by studying the production of charmed hadron due to ν_μ interactions. Charmed Hadrons have similar masses and a lifetime similar to that of τ leptons. Thus, they are considered as one of the most possible contamination background resources in the OPERA experiment.[23] Given the similar topology of ν_μ charged current events that result in charmed hadron in the final state and ν_τ charged current events, the study is powerful in assessing the capability of τ decays detection.

In paper [23] a procedure for short-lived particle detection and selection criteria was described. Selected data sample contained 50 candidate muon neutrino interaction events which were collected in 2008, 2009, and 2010 runs. The data provided the track-lines coordinates of daughter particles, interaction vertices coordinates, and impact parameters of the daughter particles. In the following, we theoretically calculate the decay lengths and tracks impact parameters

23. N. Agafonova et al., "Procedure for short-lived particle detection in the OPERA experiment and its application to charm decays," *European Physical Journal C* 74, no. 8 (2014): 1–9, ISSN: 14346052, https://doi.org/10.1140/epjc/s10052-014-2986-0.

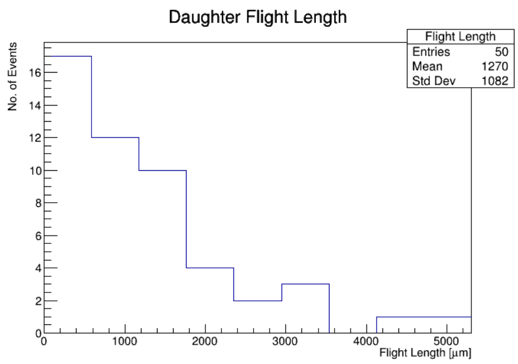
for charmed hardons produced in ν_μ charged current events. After that we compare our results of impact parameters with the ones provided in the data set. A C++ program was developed with the aid of ROOT framework for visualization of the data. The Flight lengths of daughter particles are calculated using,

$$\text{Flight Length} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

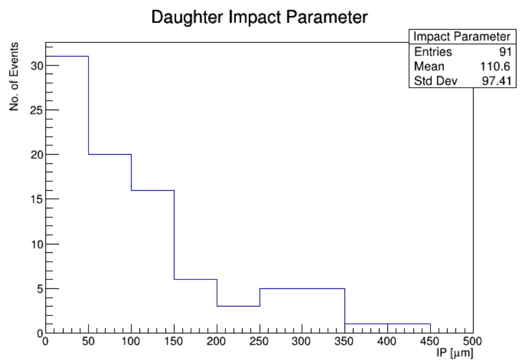
Additionally, we use the area of associated parallelogram to calculate the impact parameter of the daughter particle as follows,

$$\text{IP} = \frac{\|\vec{V_0V_1} \times \vec{V_1V_2}\|}{\|\vec{V_1V_2}\|} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \end{vmatrix}}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}$$

Fig 7b and 7a shows the results obtained for the flight lengths and impact parameters of daughter particles.



(a) Flight lengths



(b) Impact parameters

Figure 7: Distributions of the flight lengths and daughter track impact parameters of charmed hadrons.

Additionally, the calculated impact parameters were compared with the ones available in the data set and some discrepancies were found. In principle, small disagreements could exist since during the

OPERA experiment emulsion data taking and analysis were performed in several different laboratories around the world. So, it could happen, for example, that the results of decay lengths and impact parameters were obtained from a lab which was different from the one where the vertex positions and daughter particle tracks were measured. However, some events show large discrepancies. An example of such event is event "234654975" where calculated impact parameters were 9.4, 25.3, 25.6, and 68.3 micrometers, while the given parameters were 6.5, 20.6, 167.5, and 73.8 micrometers, correspondingly. Upon justification of these events, Dr. Sergey made a simple Monte Carlo simulation of 1 million events similar to the event "234654975" but with vertex and track positions randomly shifted by $\pm 5\mu$m from their original positions. The results of the simulation are shown in Fig 8. This indicates that the first, second, and fourth parameters have acceptable deviations since they lie within the simulated distributions. While the third parameter lies far ($>$ 30 standard deviations) away from the mean value of the corresponding distribution. Guided by these results, we tagged all similar events that show an error percentage of more than 30%. Table 2 shows 12 events with such discrepancies. Additionally, Two events have missing data of daughter particles. These data need further investigation which is beyond the scope of this project.

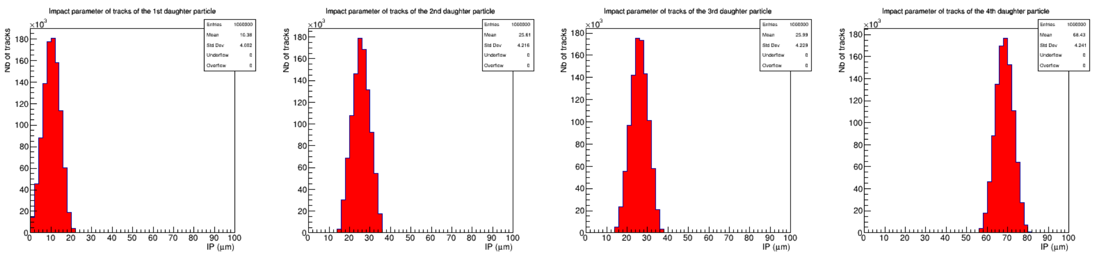


Figure 8: Simulated distributions of impact parameters for Monte Carlo events similar to event 234654975.

Table 2: Discrepancies of the impact parameters found in the data set of charmed hadrons. Note: one of the events (with MD=Y) has 1 missing value of the impact parameter, while the other one has 3 more values.

t

Event ID	Calculated IPs	Given IP	MD
222274169	"128" "107.3" "132"	"202.5" "315.5" "23.2"	N
9315114545	"26.6"	"130.7"	N
9273029609	"106.3" "106.6" "48.2"	"214.6" "327"	Y
228563573	"58.1" "15"	"9.6" "116.847"	N
9318073896	"31.4"	"403.8"	N
234654975	"6.5" "20.6" "167.5" "73.8"	"9.4" "25.3" "25.6" "68.3"	N
228197639	"3.9" "9.9"	"5.6" "47.3"	N
231012915	"75.5" "186.4"	"449.9" "197.9"	N
9248074251	"343.2"	"51.7"	N
10254046659	"139.4"	"409.9"	N
9291027303	"316.9"	"42.6" "0.5" "63.4 " "76.1"	Y
10269013559	"70.7" "320.1"	"192.1" "183.6"	N

4.2 Analysis of charged hadron multiplicities in CC

Multiplicity distribution of charged hardons reflects the dynamics of the interaction. Thus, it must be considered while studying hard scattering processes.[24] In this task, we analyze the multiplicities of charged hardons produced in the charged current ν_μ interaction events. The data set, obtained from the CERN Open Data Portal, contains 817 ν_μ interactions with the lead target where a muon was reconstructed in the final state. The multiplicity distribution extracted from the data are shown in Fig 9. Additionally, distribution of XZ and YZ track angles of the produced muons (with respect to the z-axis) plotted in a 2D histogram is shown in Fig 10. Please note, that the number of entries of the 2D histogram is 818, because the data sample includes a rare event ("11093039862") with a dimuon topology.

4.3 Visualization of OPERA ν_τ-candidate event topologies

This task aims to visualize events from the OPERA ν_τ-candidate sample where the tau lepton decay topology was reconstructed in the nuclear emulsion detectors.A web-application for 3D visualization of OPERA events from the ν_τ appearance data set has been implemented using a JavaScript (JS) THREE.js graphics library. Positions of tracks as well as the primary and secondary interaction vertices reconstructed in nuclear emulsions were inserted into the corresponding JavaScript data structures of the program. All components of HTML/CSS files were coherently implemented and a missing parts of a JS function called "MgrDraw3D-funcAdd.js" was restored in order to display the characteristic topologies of the ν_τ events in a browser window. Some

24. N. Agafonova et al., "Study of charged hadron multiplicities in charged-current neutrino–lead interactions in the OPERA detector," *European Physical Journal C* 78, no. 1 (2018): 1–8, ISSN: 14346052, https://doi.org/10.1140/epjc/s10052-017-5509-y, https://doi.org/10.1140/epjc/s10052-017-5509-y.

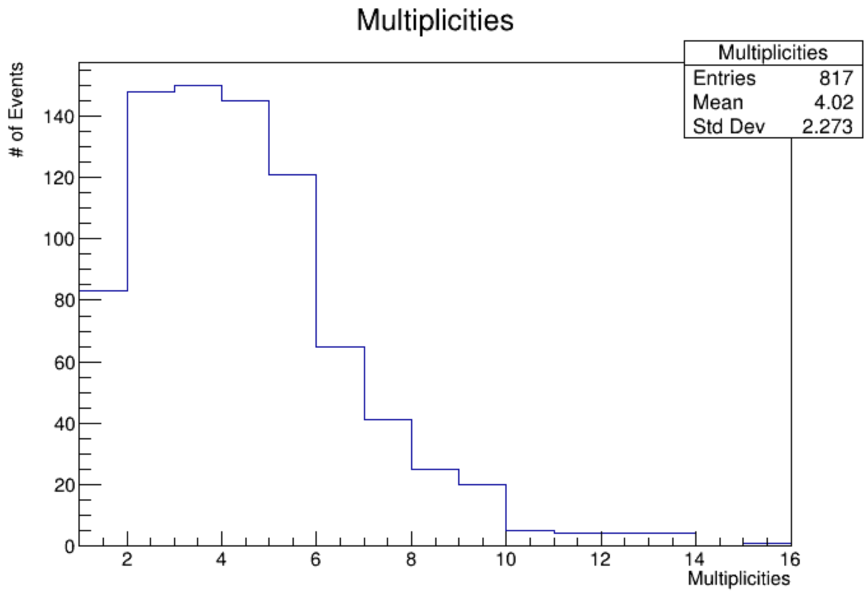


Figure 9: Multiplicities of charged hadrons in ν_μ events where a muon was reconstructed in the final stage.

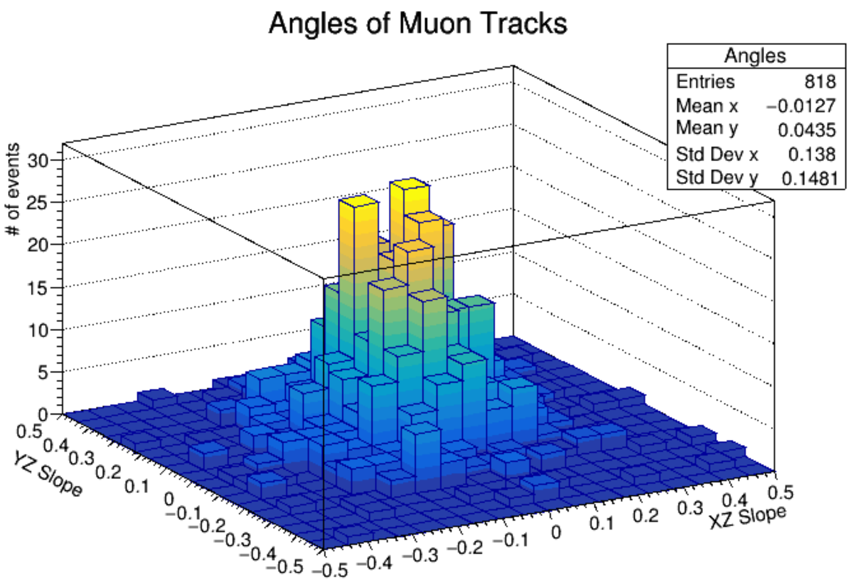
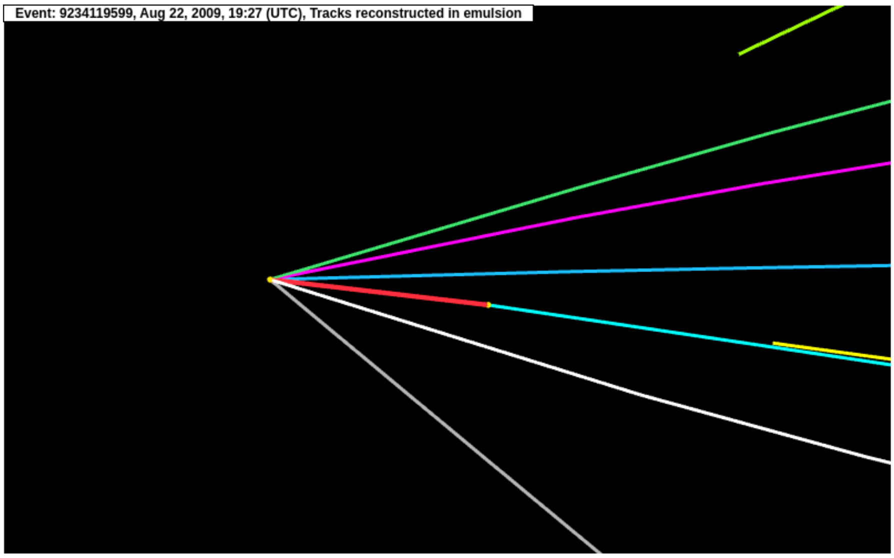
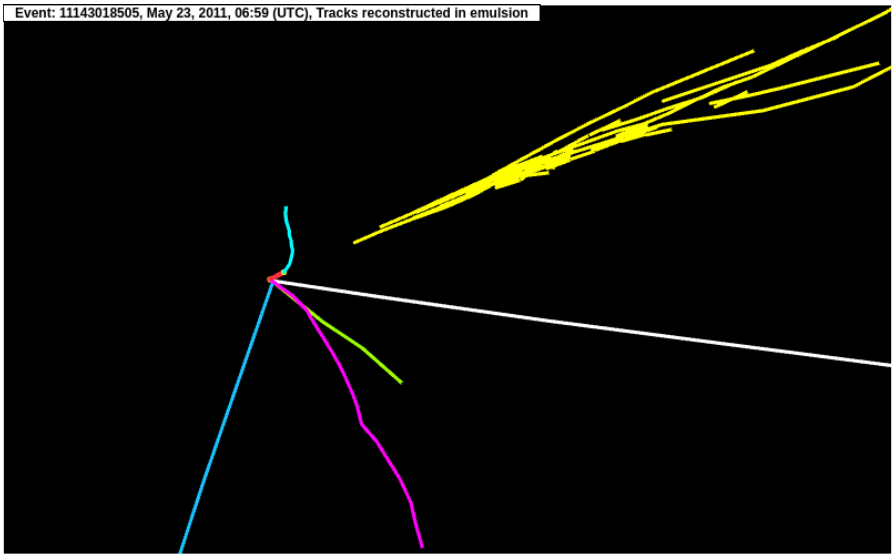


Figure 10: Distribution of reconstructed muon track angles with respect to the z-axis.

3D visualizations are shown in fig 11.



(a) Sideways visualization of event 9234119599.



(b) Head on visualization of event 11143018505.

Figure 11: Visuliztion of τ neutrino events in the OPERA experiment using 3D viewer.

5 Conclusion

The successful completion of the tasks of the project made it possible to better understand how to find and read datasets from CERN Open Data Portal and how to analyze the features of different neutrino event topologies. The implemented program codes and the developed 3D event display can be used, for example, as templates for future scientific analysis applications.

The obtained histograms have been compared with the ones from the original OPERA papers and found to be in a good agreement with the published results. However, we have found several big discrepancies in some OPERA data variables available on the Open Data Portal. More precisely, disagreement between some daughter track impact parameters provided in the emulsion dataset used for neutrino-induced charmed hadron production studies and the ones calculated by us using the measured track and vertex positions of the corresponding events can not be explained by small possible discrepancies that could be obtained in different laboratories participated in the OPERA analysis. The found discrepancies to be discussed with the relevant analysis group of the OPERA Collaboration.

6 Acknowledgment

I sincerely thank Dr. Sergey for this tremendous help in the project. Additionally, I thank him for verification of discrepancies in the events and making the simulation program. Furthermore, I would like to thank the organizing committee of INTEREST for giving me this great opportunity.

References

Acquafredda, R., T. Adam, N. Agafonova, P. Alvarez Sanchez, M. Ambrosio, A. Anokhina, S. Aoki, et al. "The OPERA experiment in the CERN to Gran Sasso neutrino beam." *Journal of Instrumentation* 4, no. 4 (2009). ISSN: 17480221. https://doi.org/10.1088/1748-0221/4/04/P04018.

Agafonova, N., A. Aleksandrov, A. Anokhina, S. Aoki, A. Ariga, T. Ariga, D. Bender, et al. "Procedure for short-lived particle detection in the OPERA experiment and its application to charm decays." *European Physical Journal C* 74, no. 8 (2014): 1–9. ISSN: 14346052. https://doi.org/10.1140/epjc/s10052-014-2986-0.

Agafonova, N., A. Aleksandrov, A. Anokhina, S. Aoki, A. Ariga, T. Ariga, A. Bertolin, et al. "Study of charged hadron multiplicities in charged-current neutrino–lead interactions in the OPERA detector." *European Physical Journal C* 78, no. 1 (2018): 1–8. ISSN: 14346052. https://doi.org/10.1140/epjc/s10052-017-5509-y. https://doi.org/10.1140/epjc/s10052-017-5509-y.

Bilenky, S. "Neutrino oscillations: From a historical perspective to the present status." *Nuclear Physics B* 908 (2016): 2–13. ISSN: 05503213. https://doi.org/10.1016/j.nuclphysb.2016.01.025. arXiv: 1602.00170. http://dx.doi.org/10.1016/j.nuclphysb.2016.01.025.

Bitter, Olivia Meredith. "An Introduction to Neutrino Oscillation : A Quantum Mechanical Perspective," 2021, 1–13.

Bussey, Peter J. *Modern Particle Physics, by Mark Thomson*, 55:251–252. 3. 2014. ISBN: 9781107034266. https://doi.org/10.1080/00107514.2014.907354.

Fantini, G., A. Gallo Rosso, V. Zema, and F. Vissani. “Introduction to the formalism of neutrino oscillations.” *The State Of The Art Of Neutrino Physics: A Tutorial For Graduate Students And Young Researchers*, 2018, 37–119. https://doi.org/10.1142/10600. arXiv: arXiv:1802.05781v2.

Giacomelli, G., M. Giorgini, L. Patrizii, and M. Sioli. “Neutrino oscillations in the atmospheric parameter region: From the early experiments to the present.” *Advances in High Energy Physics* 2013 (2013). ISSN: 16877357. https://doi.org/10.1155/2013/464926.

Mondal, Spandan. “Physics of Neutrino Oscillation,” 2015. arXiv: 1511.06752. http://arxiv.org/abs/1511.06752.

Naumov, D. V. “Introduction to neutrino physics.” *Proceedings of the International Baikal Summer School on Physics of Elementary Particles and Astrophysics 2010*, 2010, 218–262. ISSN: 0029-0181. https://doi.org/10.1007/978-981-287-715-4_1.

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