Puzzles of Multiplicity

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Abstract

Multiplicity stands as a ubiquitous subject of particle physics. In particular, it has been a tool to provide us with a way to understand matter at a more fundamental level in the context of strong interactions. Due to this, it has been of great interest to create theoretical models to explain complex behavior that occurs at high energy levels. In this project, the multiplicity distribution in the context of e^+e^- annihilation was modeled by using Markov branching processes and probability generating functions to represent quark and gluon jets. We showed the reliability of this model's predictive power using minimizing algorithms. This approach enables a probabilistic description of multiplicity distributions of partons.

Contents

Chapter 1

Introduction

Multiplicity is defined as the number of particles that arise from the interactions within a collision or a decay. Because quarks and gluons cannot exist as free particles, the evolution from an initial partonic interaction to the final detectable hadrons involves processes like parton showering and hadronization. The multiplicity distribution encodes information about underlying QCD dynamics. Measuring parton jet multiplicities provide as stringent test of perturbative QCD predictions.

Properties of multiplicity distribution carry important physical insight. The mean multiplicity is the average number of secondary particles produced in a class of events. The difference in particle number in an event to event basis is measured as its variance. These parameters help reveal the mechanisms governing particle production and the degree of complexity in the interaction.

Within the framework of QCD, the theory describing the strong interaction, the early stages of a high-energy collision can be calculated at the level of partonic amplitudes. This perturbative regime called pQCD-successfully describes hard scattering processes involving large momentum transfer, where quarks and gluons behave as nearly free particles due to asymptotic freedom. A major challenge arises when describing the transition from these energetic partons to the hadrons that are ultimately detected. At low energies, QCD becomes strongly coupled, and confinement prevents quarks and gluons from existing as isolated particles. This non-perturbative stage, known as hadronization, cannot yet be derived from first principles in QCD.

Chapter 2

QCD Jets

2.1 QCD Jets as Markov Branching Processes

Giovannini's work showed that we can use Markov branching processes to QCD to describe particle collisions as a sequence of random branching events. To study multiplicity at high-energy collisions, we can consider the QCD evolution parameter as a function of energy:

$$Y = \frac{1}{2\pi b} \ln(1 + b\alpha \ln \frac{Q^2}{\mu^2})$$

This tells us about the thickness of jets which refers to the spread of a jet is a variable that explains the variance of the particles in a jet. This also provides insight into the dynamics of hadronization and parton showering. The three elementary processes that can explain parton distribution are:

- $g \to g + g$ Gluon fission with a transition probability of $A\Delta$
- $g \to g + q$ Quark Bremsstrahlung with a transition probability of $\tilde{A}\Delta$
- $g \to \bar{q} + q$ Quark pair creation with a transition probability of $B\Delta$

This is under the assumption that the probability coefficients are independent of Y and are constant.

We can define the probability generating functions of parton evolution through Y as follows:

$$G(u_g, u_q, Y) = \sum_{n_g, n_q = 0}^{\infty} P_{1,0;n_g,n_q}(Y)\, u_g^{n_g}\, u_q^{n_q}$$

$$Q(u_g, u_q, Y) = \sum_{n_g, n_q=0}^{\infty} P_{0,1;n_g,n_q}(Y)\, u_g^{n_g}\, u_q^{n_q}$$

Where each probability generating function defines how a single gluon or a single quark transitions to n_g gluons or n_q quarks. Noting event independence, combining these functions gives a general function for the process.

$$\sum_{n_g, n_q=0}^{\infty} P_{m_g,m_q;n_g,n_q}(Y)\, u_g^{n_g}\, u_q^{n_q} = [G(u_g, u_q; Y)]^{m_g}\, [Q(u_g, u_q; Y)]^{m_q}\,.$$

To compute parton transition over a period of time we use the probability identity called the Chapman-Kolmogorov equation. This can be obtained simply by integrating through all possible states of the function.

one obtains a set of coupled differential equations describing how the generating functions evolve when the *initial* parton undergoes a splitting in an infinitesimal interval ΔY. This leads to the backward Kolmogorov equations

$$\frac{\partial G}{\partial Y} = -A\, G + A\, G^2 - B\, G + B\, Q^2.$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}\, Q + \tilde{A}\, QG.$$

Here A is the gluon fission rate $g \to gg$, B is the quark–antiquark creation rate $g \to q\bar{q}$, and $\tilde{A}$ is the quark bremsstrahlung rate $q \to qg$.

A complementary formulation can be derived directly from the evolution of the transition probabilities. Expanding $P_{1,0;n_g,n_q}(Y + \Delta Y)$ to first order in ΔY and enforcing probability conservation yields the forward Kolmogorov equations,

$$\frac{\partial G}{\partial Y} = A\, G^2 - A\, G - B\, G.$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}\, Q + \tilde{A}\, QG.$$

2.2 Gluon jet

For a jet initiated by a single gluon, the probability that the number of gluons remains n_g at evolution parameter Y satisfies

$$P_{1,0;n_g}(Y) = e^{-AY} \left(1 - e^{-AY}\right)^{n_g - 1}, \qquad n_g \geq 1.$$

This is a geometric-like distribution. The mean gluon multiplicity grows exponentially,

$$\langle n_g \rangle = e^{AY},$$

and the variance follows

$$D^2 = e^{AY} \left(e^{AY} - 1\right).$$

Quark jet

For a quark-initiated jet, the solution takes a Polya–Eggenberger form. Defining $\mu = \tilde{A}/A$, the probability of having n_g gluons at scale Y is

$$P_{0,1;n_g}(Y) = \frac{\mu(\mu+1)\cdots(\mu+n_g-1)}{n_g!} \, e^{-\tilde{A}Y} \left(1 - e^{-AY}\right)^{n_g},$$

and the mean number of emitted gluons is

$$\langle n_g \rangle = \mu \left(e^{AY} - 1\right).$$

The variance is

$$D^2 = \mu \, e^{AY} \left(e^{AY} - 1\right).$$

Hadronization

In the two-stage model, the partonic cascade is followed by a phenomenological hadronization step. The final hadron multiplicity distribution is written as a convolution between partonic multiplicities and the hadronization probability,

$$P_n(s) = \sum_{n_g} P_{n_g} \, P_{\text{had}}(n_g, s),$$

where P_{n_g} is given by the partonic branching model above.

In the regime considered, the partonic stage is described by a Polya–Eggenberger (negative-binomial–type) distribution, yielding

$$\frac{\sigma_{n_g}}{\sigma_{\text{tot}}} = \frac{\mu(\mu+1)\cdots(\mu+n_g-1)}{n_g!}\left(\frac{\langle n_g\rangle}{\langle n_g\rangle+\mu}\right)^{n_g}\left(\frac{\mu}{\langle n_g\rangle+\mu}\right)^{\mu}.$$

This distribution provides a good description of the observed charged multiplicity in e^+e^- annihilation data, supporting the two-stage interpretation where the partonic cascade determines the shape and hadronization redistributes it into final hadrons.

Chapter 3

Results

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3.1 Parameters obtained with fitting

$\sqrt{s}$	Ω	k_p	$\bar{m}$	$\bar{n}^h$	N	α	χ^2
14	1.998 ± 0.034	16.000 ± 2.074	0.084 ± 0.064	4.465 ± 0.095	27.724 ± 10.242	0.965 ± 0.236	2.799
22	1.999 ± 0.035	3.170 ± 2.628	1.959 ± 0.759	4.675 ± 0.302	27.799 ± 14.646	0.214 ± 0.075	1.663
34.8	1.998 ± 0.015	7.530 ± 1.762	11.148 ± 5.456	3.972 ± 0.238	15.000 ± 5.466	0.128 ± 0.050	8.849
43.6	2.004 ± 0.029	40.006 ± 9.593	34.925 ± 2.822	1.174 ± 0.170	7.021 ± 2.587	0.311 ± 0.051	5.865

Table 3.1: Measurements with uncertainties.

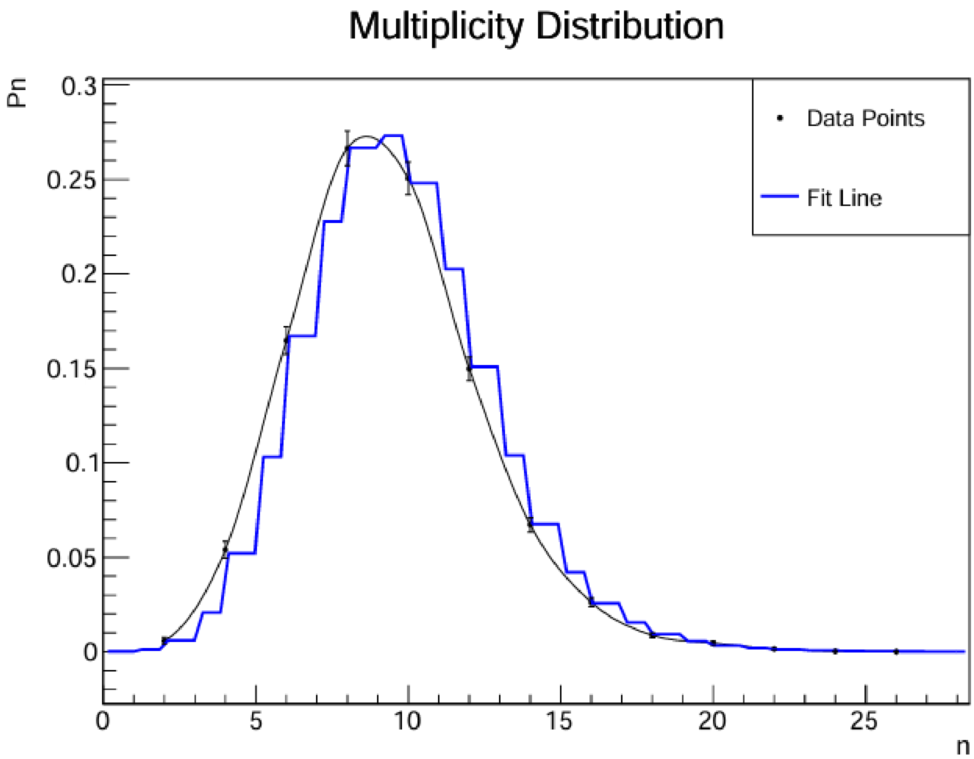

Figure 3.1: Energy level 14 GeV

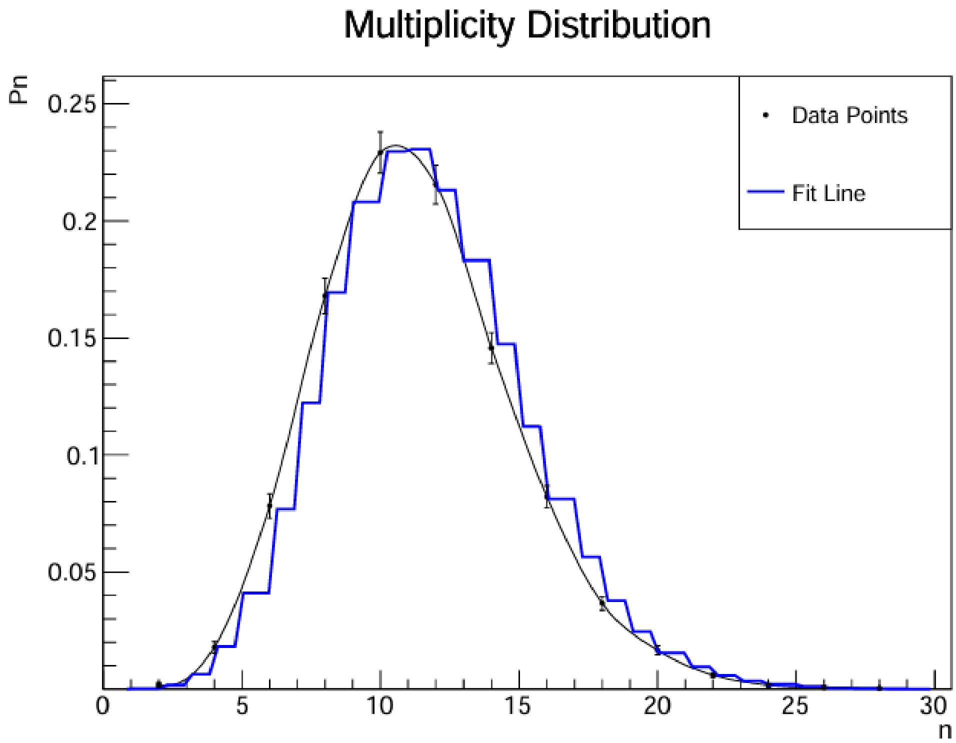

Figure 3.2: Energy level 22 GeV

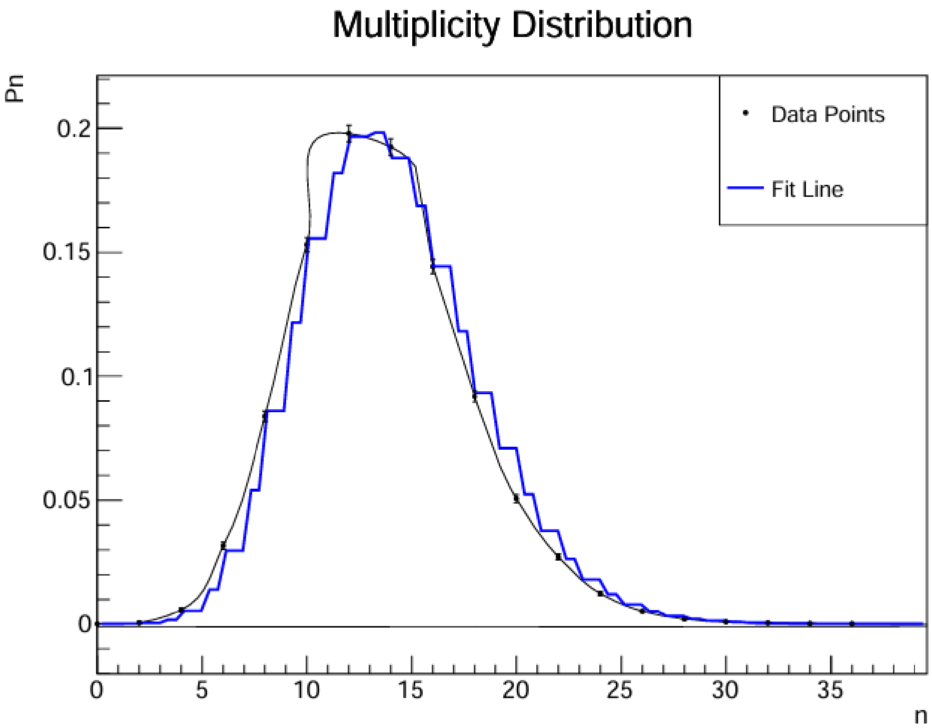

Figure 3.3: Energy level 34.8 GeV

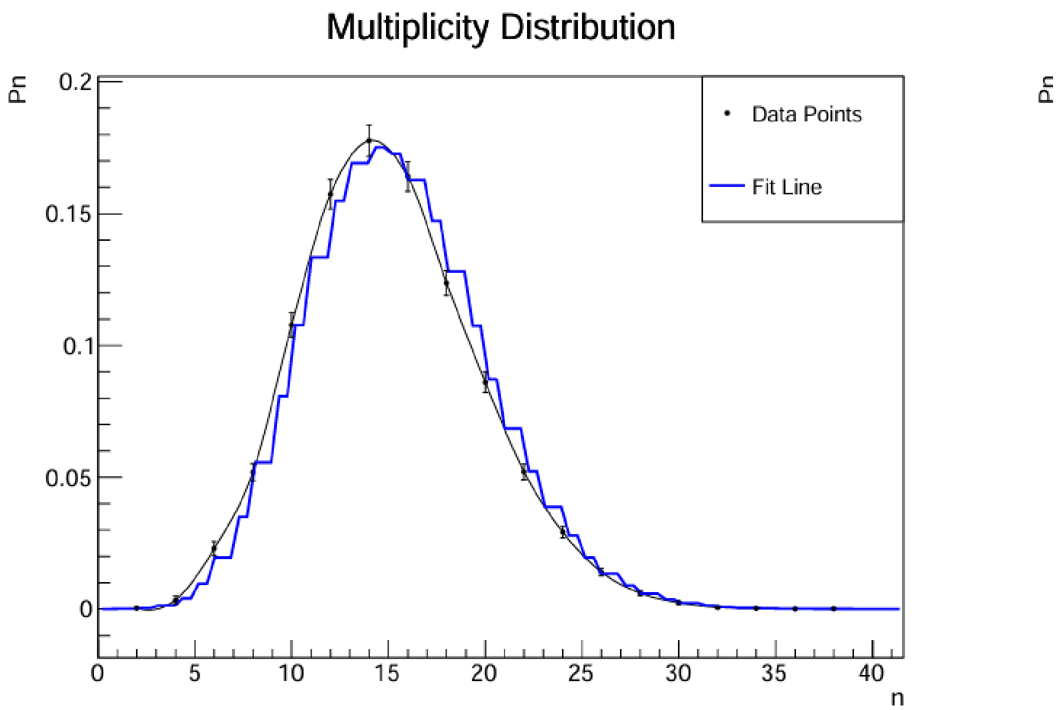

Figure 3.4: Energy level 43.6 GeV

Chapter 4

Conclusions

The study of multiplicity in in high-energy electron–positron annihilation offers important insights into strong interactions but becomes challenging at higher energies as inelastic channels grow. To address these difficulties, we applied statistical methods using the Two Stage Model, which treats QCD jets as Markov branching processes and provides a probabilistic description of parton showers and their multiplicity distributions. Additionally, we developed C++ code to model the charged particle multiplicity distribution at multiple energy levels, finding good agreement between the TSM predictions and experimental data. Overall, the results demonstrate that the TSM offers a reliable and effective framework for modeling multiparticle production at high energies.

Bibliography

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