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FINAL REPORT ON THE INTEREST PROGRAMME

Demonstration of negative differential resistance in the IV curve of φ_0 Josephson junction

Supervisor:
Dr. Majed Nashaat AbdelGhani

Student:
George Krugan, Russia
NRU MPEI

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Abstract.

This study presents calculations of IV-characteristics for superconductor-insulator-superconductor point contact Josephson Junction with different values of dissipation parameter. We used fourth order Runge Kutta numerical method to solve a system of two Josephson's equations. Also we consider superconductor-ferromagnetic-superconductor point contact Josephson Junction with different values of parameters. In this case to find the dynamics for magnetic moment in ferromagnetic layer we solve Landau-Lifshitz-Gilbert equation. The results show an area of negative differential resistance in the IV curve of the SFS junction, resulting in an additional locking step in the maximum value of magnetic moment. This is due to the spin orbit interaction which couples the Josephson phase and magnetic moment in the SFS Junction.

1. Introduction

The phenomena of Josephson effect was found in 1962 by Brian David Josephson. He predicted two main equations of the effect for current and voltage with Bardeen-Cooper-Schrieffer theory of superconductivity.

The effect is observed if we consider two superconductors weakly connected by an electrical contact. Such contact can be established in many different ways. Prominent examples are tunneling barriers, point contacts, superconductor-ferromagnetic-superconductor (SFS) junction or insulating layers connecting the two superconducting electrodes (SIS junction). It named *Josephson junction* (JJ).

Two Josephson' s equations for current (1) and voltage (2) are used to describe the dynamics of the Josephson phase:

$$I = I_c \cdot sin(\varphi) \tag{1}$$

$$V \propto \frac{d\varphi}{dt}, \tag{2}$$

where φ is phase difference between current phases of two superconductors and I_c is a critical current.

The equation (1) can be received from the following considerations.

Let we first speculate what determines the supercurrent between two weakly connected superconductors. It certainly can depend on the Cooper pair densities $|\psi_1|^2 = n^\star_{s,1}$ and $|\psi_2|^2 = n^\star_{s,2}$ in the junction electrodes. However, since the coupling between the two superconductors is weak and, hence, the supercurrent density between them is small, we can assume that the supercurrent density between the two junction electrodes is not changing $|\psi|^2$. However, although the amplitude of the wave functions in the electrodes does not play a role, the supercurrent density certainly is expected to depend on the phase of the wave functions. If we write down equation for probability current density of quasiparticles in superconductor and accept that it should be gauge-invariant, we can receive:

$$j = \frac{q^* n_s^* \hbar}{m^*} \gamma(\boldsymbol{r}, t), \tag{3}$$

where $\gamma(r, t)$ is a gauge-invariant phase difference φ. From (3) we can expect that $J_c \propto \varphi$ and $J_c(\varphi)$ is a 2π periodic function. Also we should mention that $J_c(0) = J_c(2\pi n) = 0$ because in the absence of any current the phase difference φ should be 0. Finally, in the case of weak coupling we can receive the first Josephson equation (1) for density of supercurrent or supercurrent itself in JJ.

To substantiate the equation (2) we should use the time derivative of the gauge invariant phase difference:

$$\frac{\partial\varphi}{\partial t} = \frac{\partial\theta_2}{\partial t} - \frac{\partial\theta_1}{\partial t} - \frac{2\pi}{\Phi_0}\frac{\partial}{\partial t}\int_1^2 \boldsymbol{A}(\boldsymbol{r}, t) dl$$

or

$$\frac{\partial\varphi}{\partial t} = \frac{2\pi}{\Phi_0}\int_1^2 \boldsymbol{E}(\boldsymbol{r},t)dl,$$

where $\boldsymbol{A}(\boldsymbol{r},t)$ is a vector-potential, Φ_0 is a quantum of magnetic field. The integral is voltage. That is why we receive the equation (2) in the following form (4):

$$\frac{\partial\varphi}{\partial t} = \frac{2\pi}{\Phi_0}V \tag{4}$$

for constant voltage. Solving (4) we obtain:

$$\varphi(t) = \varphi_0 + \frac{2\pi}{\Phi_0}Vt.$$

Then, the Josephson current $I = I_c \cdot sin(\varphi(t))$ oscillates with Josephson frequency:

$$\omega = \frac{2\pi}{\Phi_0}V. \tag{5}$$

Thus, we have a describing of the Josephson effect. [1]

2. Results

2.1 IV characteristics for single Josephson junction with insulator barrier

To describe the point contact short JJ, we used the resistively capacitance shunted junction RCSJ model [1,2]. Initially, there is only supercurrent in JJ below critical current. After heating or increasing of the current it is appeared the normal current which is proportional to voltage through resistance. Finally, there is a displacement current which provides hysteresis in IV-characteristic because JJ behave like capacitor. The short JJ can be described like RCSJ (Fig.3) model with resistor, capacitor and supercurrent because the current always chooses the path of least resistance.

In this case we need to use the first and the second Josephson equations (1), (2) and with some transformations the equation for RCSJ model looks like the following form [1,2]:

$$I = I_c sin\varphi + C\frac{d^2\varphi}{dt^2} + \frac{1}{R}\frac{d\varphi}{dt}, \tag{6}$$

where R is a normal resistance, C is a capacitance. [3]

Then with the solving of system of differential equations (4) and (6) in the dimensionless form we have:

$$I = sin\varphi + \frac{dV}{dt} + \beta\frac{\partial\varphi}{\partial t}.$$

where the current is normalized to the critical current of the JJ, and the time is normalized to the inverse of characteristic frequency of the JJ ($\omega_c = 2eI_cR/\hbar$), and β is the dissipation parameter (β) which is given by $\beta = \frac{1}{R}\sqrt{\frac{\hbar}{2eI_cC}}$.

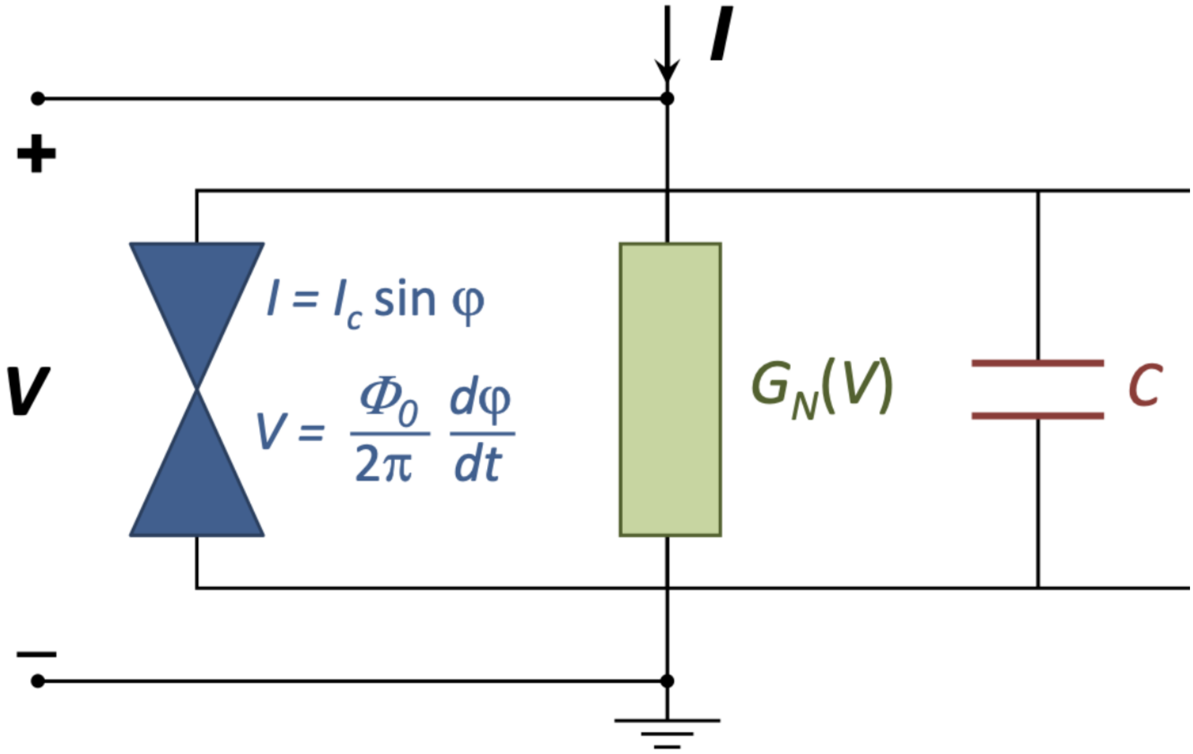


Fig.3. RCSJ model for JJ. This figure is taken from [1]

One of the goals of the practice is studying the effect of the dissipation parameters on the IV characteristics of JJ.

In Fig. (4) we demonstrate different IV curves with different β. Cases with β<1 are called «underdamped» and «overdamped» for cases with β>1. For underdamped case the IV curve demonstrate a hysteresis region, which decreases by increasing the dissipation parameters.

In the plots (Fig.4.a) and (Fig.4.b) the zero voltage state appears) before the critical current (I=1). Then the junction demonstrate resistive region with slope increases by decreasing the dissipation parameter. This is because β is inversely proportional to the resistance of the JJ. While by decreasing the current from Imax to zero, the IV may demonstrate hysteresis area due to the effect of the junction capacitance. In this case we have underdamped junction. While if the capacitance of the junction is neglected the IV curve for both current directions (increasing and decreasing) coincides and no hysteresis appears.

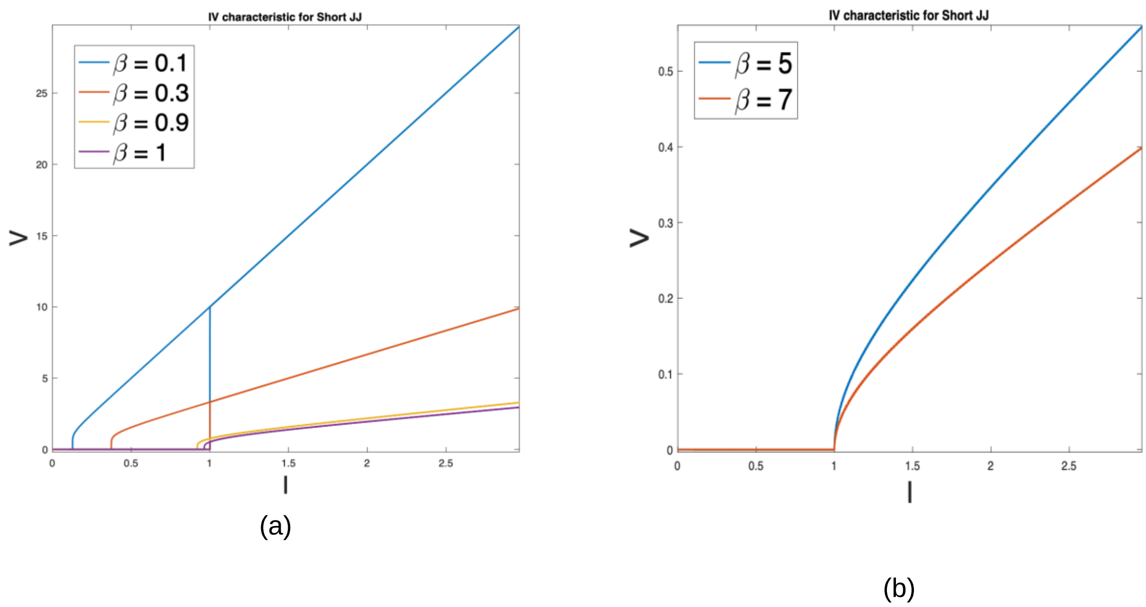


Fig.4. IV-characteristics for different values of the dissipation parameters

The IV-characteristic for the Josephson junction shows steps at V = nħω/2e (n is integer) when it is subjected to a microwave radiation with frequency ω and amplitude I_{ac}. These steps, caused by absorption of "n" quanta of electromagnetic radiation at the Josephson frequency. These steps are referred to Shapiro steps [2]. Fig.5 show the appearance of Shapiro step in the IV curve at V=ω=0.4. The appearance of these steps at fixed voltages has been predicted by B. Josephson and was explained due to the formation of higher harmonics of the signal frequency due to the nonlinearity of the Josephson junction. We may also have n^{th} steps which corresponds to the phase locking of the junction oscillation by this n^{th} harmonic.

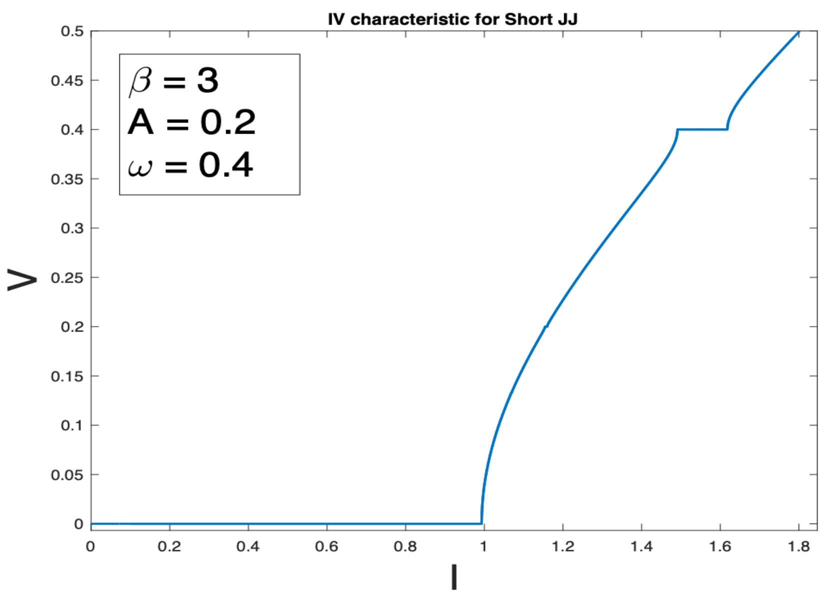


Fig.5. IV-characteristic for external radiation effect

2.2 IV characteristics for single Josephson junction with ferromagnetic barrier

The second part of the practice is devoted to φ_0 Josephson junction where an insulator layer is replaced by ferromagnetic layer. We studied the effect of spin-orbit coupling, especially, under external radiation.

Josephson junctions may exhibit a negative differential resistance in the current-voltage characteristics, which plays an essential role in many applications, in particular for THz radiation emission. In Ref. [4] the authors consider φ_0JJ and demonstrate an important role of the states with negative differential resistance in the locking of magnetization and Josephson oscillations at ferromagnetic resonance.

In φ_0 Josephson junction the role of spin-orbit coupling in the appearance of the nonlinearity in the IV curve and additional steps at small radiation amplitudes are demonstrated. It shows that in junctions with a strong coupling the states with negative differential resistance result in an additional step with corresponding oscillations having the same frequency as the oscillations at the first step, but a different amplitude and different dependence on the radiation frequency. This opens a unique way to control not only the frequency but also the amplitude of the magnetic precession in hybrid superconducting systems. [4]

If we want to plot the IV for SFS JJ one need to couple the Josephson equations with the Landau-Lifshitz-Gilbert equation [4]:

$$\frac{d\boldsymbol{M}}{dt} = -\gamma \boldsymbol{M} \times \boldsymbol{H_{eff}} + \frac{\alpha}{M_0}\left(\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}\right), \tag{7}$$

where $\boldsymbol{H_{eff}} = \frac{K}{M_0}\left[Grsin\left(\varphi - \frac{M_y}{M_0}\right)y + \frac{M_z}{M_0}z\right]$ and

α is a Gilbert damping parameter. [5]

The system (7) with Josephson equations splits to the following equations in the dimensionless form [4]:

$$\dot{m}_x = \frac{\omega_F}{1+\alpha^2}\{-m_y m_z + Grm_z sin(\varphi - rm_y) - \alpha[m_x m_z^2 + Grm_x m_y sin(\varphi - rm_y)]\}$$

$$\dot{m}_y = \frac{\omega_F}{1+\alpha^2}\{m_x m_z - \alpha[m_y m_z^2 - Gr(m_z^2 + m_x^2)sin(\varphi - rm_y)]\}$$

$$\dot{m}_z = \frac{\omega_F}{1+\alpha^2}\{-Grm_x sin(\varphi - rm_y) - \alpha[Grm_y m_z sin(\varphi - rm_y) - m_z(m_x^2 + m_y^2)]\}$$

$$\frac{dV}{dt} = \frac{1}{\beta_c}\left[I + Asin(\omega_R t) - V - sin(\varphi - rm_y) + r\frac{dm_y}{dt}\right]$$

$$\frac{d\varphi}{dt} = V,$$

where $\beta_c = \frac{1}{\beta^2} = \frac{2eI_cCR^2}{\hbar}$ is the McCumber parameter; φ is the phase difference between the superconductors across the junction; $G = \frac{E_J}{K\nu}$; K is an anisotropic constant; ν is the volume of ferromagnetic layer; $r = l\upsilon_{so}/\upsilon_F$ is parameter of spin-orbit coupling; it characterizes a relative strength of spin-orbit interaction; υ_F is Fermi velocity; $l = \frac{4hL}{\hbar\upsilon_F}$; L is the length of the ferromagnetic layer; h denotes the exchange field in the ferromagnetic layer; α is a phenomenological damping constant; $m_i = M_i/M_0$ for $i = x, y, z$; $M_0 = ||\boldsymbol{M}||$; $\omega_F = \Omega_F/\omega_c$ with the ferromagnetic resonance frequency $\Omega_F = \gamma K/M_0$; γ is the gyromagnetic ratio; characteristic frequency $\omega_c = 2eRI_c/\hbar$; A is the amplitude of external radiation normalized to I_c; and ω_R is the frequency of external radiation normalized to ω_c. Here time is normalized in units of ω_c^{-1}, external current I in units of I_c, and voltage V in units of $V_c = I_cR$.

2.3.Demonstration of the negative differential resistance in the IV curve

Fig. 6 shows the IV with external radiation frequency equal to ferromagnetic resonance frequency. In this case we can see harmonics (0.5) and subharmonics (0.25) and area on the curve between them corresponds to the negative differential resistance. This is very interesting and important thing, which has lots of applications in the industry.

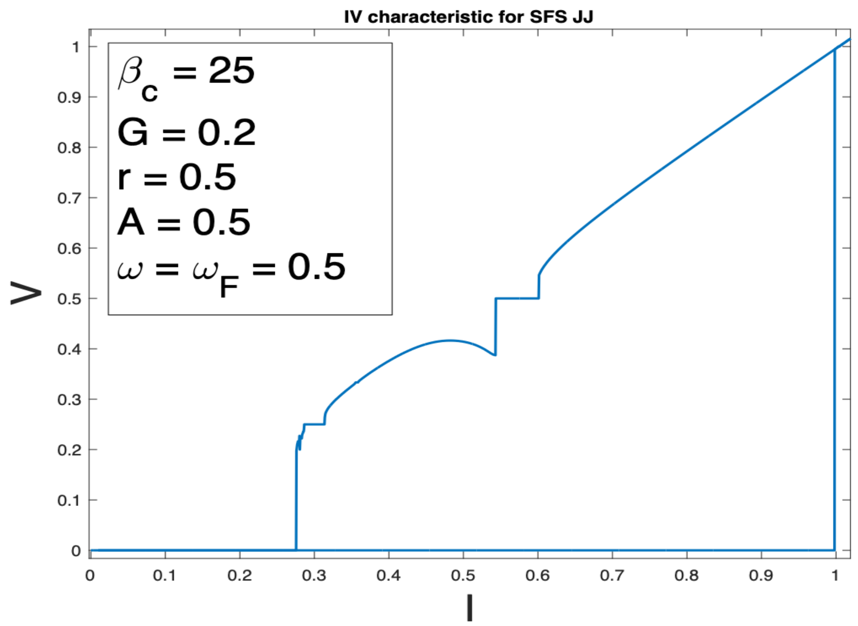
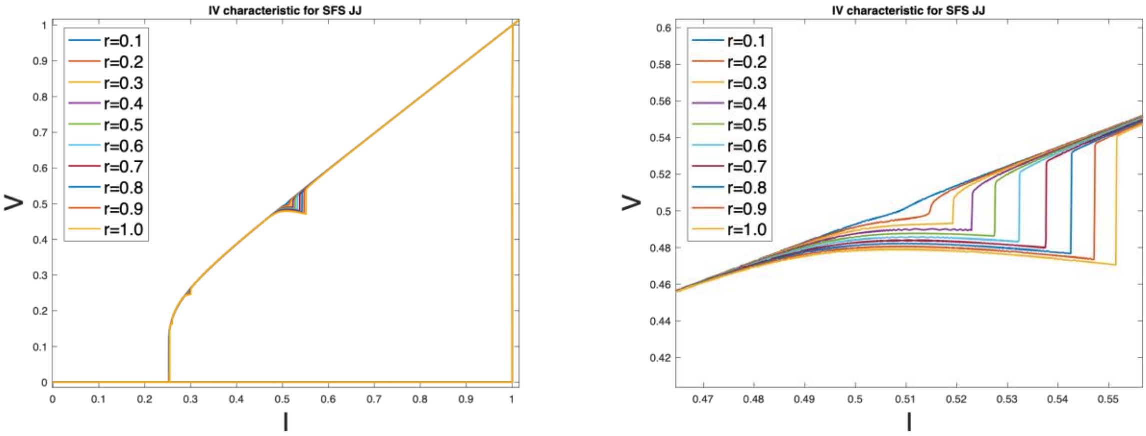


Fig.6. IV-characteristic for SFS JJ with external radiation

Fig. 7 shows us the effect of spin-orbit coupling parameter r, which affects the area with negative differential resistance, which is located near ω_F. With increasing of r the area of negative differential resistance increases. With an increase in the spin-orbit coupling, the nonlinear region presented in Fig. 7.b is going down. Thus the appearance of the Shapiro Steps within the resonance branch can be observed for a certain range of the r parameter. [4]



(a) (b)

Fig.7. IV-characteristic for SFS JJ without external radiation
($G = 0.01$, $\alpha = 0.01$, $\omega_F = 0.5$)

Fig. 8 and Fig. 9 are pretty similar, but there are different sets of values of r. In (Fig. 8.a) (Fig. 9.a) we can see two interesting areas: the first one (Fig. 8.b) (Fig. 9.b) is located near ω_F (harmonic) and the second one (Fig. 8.c) (Fig. 9.c) — near $\frac{\omega_F}{2}$ (subharmonic). It is Shapiro steps. The first one is an area of negative differential resistance for $r = 0.5$ and larger. The same picture we can see on (Fig. 9.b) for other values of r. There is a hump (dome) on the plots as a manifestation of the ferromagnetic resonance, and the second Shapiro step appeared in the part of the resonance branch with negative differential resistance. We emphasize that the negative differential resistance state leads to a unique situation when two Shapiro steps in the IV characteristic coexist at the same frequency. [4]

The second one (Fig. 8.c) and (Fig. 9.c) shows an enlarged part of the IV in the subharmonic step “V=ω/2”, we only see nonlinear region without the negative differential resistance region. In the (Fig. 8.d) and (Fig. 9.d) we can see some other subharmonic and fractional steps.

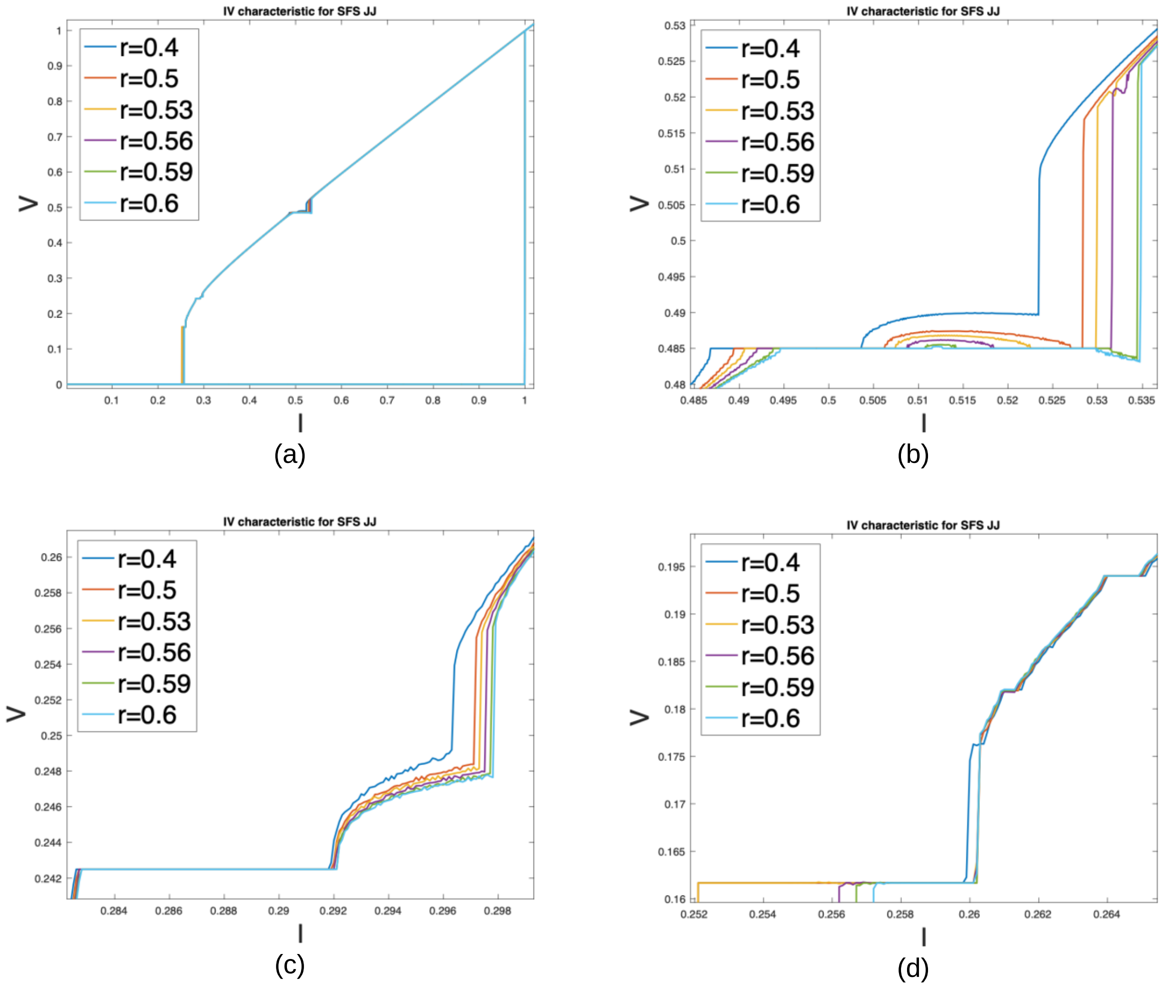


Fig.8. IV-characteristic for SFS JJ with specific external radiation ($G = 0.01$, $\alpha = 0.01$, $A = 0.1$, $\omega_R = 0.485$, $\omega_F = 0.5$)

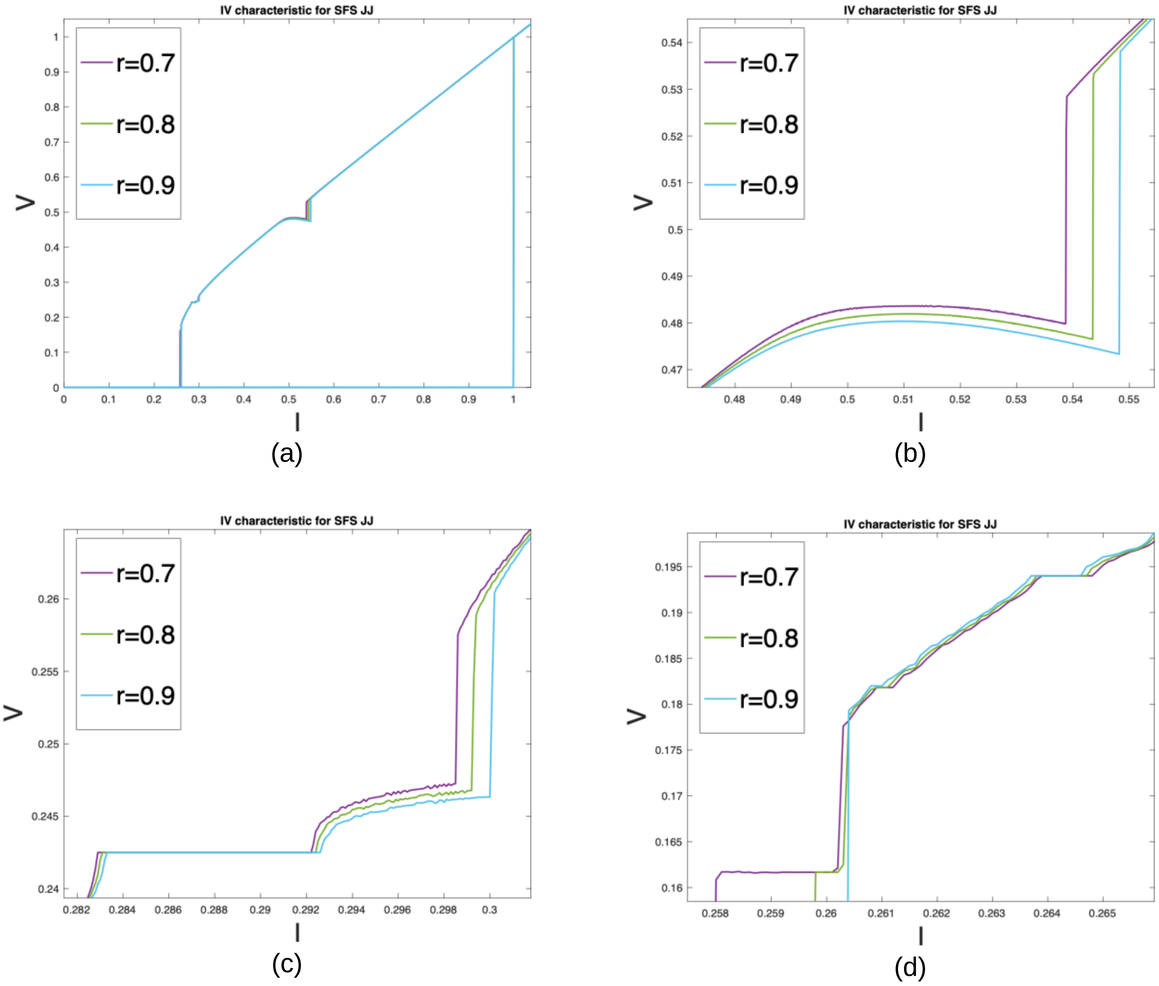


Fig.9. IV-characteristic for SFS JJ with specific external radiation
(G = 0.01, α = 0.01, A = 0.1, ω_R = 0.485, ω_F = 0.5)

3. Conclusion

We studied the IV-characteristics for superconductor-insulator-superconductor point contact Josephson Junction at different values of dissipation parameter. We show that the IV curve demonstrates hysteresis region in the underdamped case, where the junction capacitance is considered. Also, we consider superconductor-ferromagnetic-superconductor point contact Josephson Junction with different values of parameters. In this case we solve Landau-Lifshitz-Gilbert equation along with Josephson equations. The results show an area of negative differential resistance in the IV curve of the SFS junction as a result of spin-orbit coupling which couples the Josephson phase and magnetic moment in the SFS Junction. We demonstrate how the appearance of the negative differential resistance changes with changing the strength of the spin orbit coupling. This may open up unique perspectives for the control and manipulation of the nonlinear regions in the IV characteristics of φ_0 Josephson junction.

4. Acknowledgment

Through this study I have learnt how to apply numerical methods to a real research topic. I have studied the phenomena of negative differential resistance and, at whole, the phenomena of superconductivity with terms of theoretical physics. I have learnt Josephson effect, effect of external radiation, magnetic properties of SFS JJ. There is a whole field of research for the phenomena of negative differential resistance. So special thanks to JINR INTEREST program for providing such program.

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