



JOINT INSTITUTE FOR NUCLEAR RESEARCH  
*Laboratory of Information Technologies*

# FINAL REPORT ON THE INTEREST PROGRAMME

## *Introduction to Quantum Computing*

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## Table of Contents

1. Abstract.....	3
2. Spin Quantum Mechanics.....	3
3. Qubit experiments.....	4
3.1 Resonance frequency scan .....	4
3.2 $\pi$ Pulse Calibration.....	5
3.3 Determining 0 vs 1.....	6
3.4 T1 Relaxation time.....	7
3.5 Precise qubit frequency measurement using a Ramsey experiment.....	8
3.6 Measuring $T_2$ using Hahn Echoes .....	9
3.7 Dynamical decoupling .....	9
4. QuLogic .....	11
5. Quantum algorithm.....	12
6. Conclusion.....	13

## 1. Abstract

Through this program, we got familiar with the basic concepts of quantum mechanics and quantum computing. We observed the properties of qubits, as the basic building blocks of quantum computers, such as spin, superposition, and entanglement. We got to perform experiments on real qubits and measure their characteristics through the IBM Q Experience platform. Additionally, we learned about quantum logic gates used for qubit state manipulation and the QASM-2 language, which we were then able to use to create simple quantum circuits and see how they work on real quantum chips as opposed to the theoretical expectation. Finally, we were able to demonstrate some of the advantages of quantum computers through the use of Grover's algorithm.

## 2. Spin Quantum Mechanics

The discovery of Quantum Mechanics took the world of physics by storm. The famous Schrödinger equation, postulated by the Austrian physicist, introduced new concepts that were at times in contradiction to our understanding of reality. The driving idea was that particles also display a wave-like nature (as shown with the double-slit experiment). Scientific and philosophical battles emerged as the world's most renowned scientists picked sides. Even Einstein disapproved of the theory, despite proving a crucial figure in its early development, stating that God does not play dice. Soon enough, nobody could disagree as experiments backed the theory 100%. The only thing that was up for debate was the interpretation of the mathematical equations.

One of the most important fundamental characteristics of nature discovered was the concept of spin. Although spin can not be derived from the already mentioned non-relativistic Schrödinger equation, it turns up as the solution of the relativistic Dirac wave equation. Nonetheless, the Stern–Gerlach experiment demonstrated that the spatial orientation of angular momentum is quantized, which could only be a result of a quantum theory, and is intrinsically tied to spin.

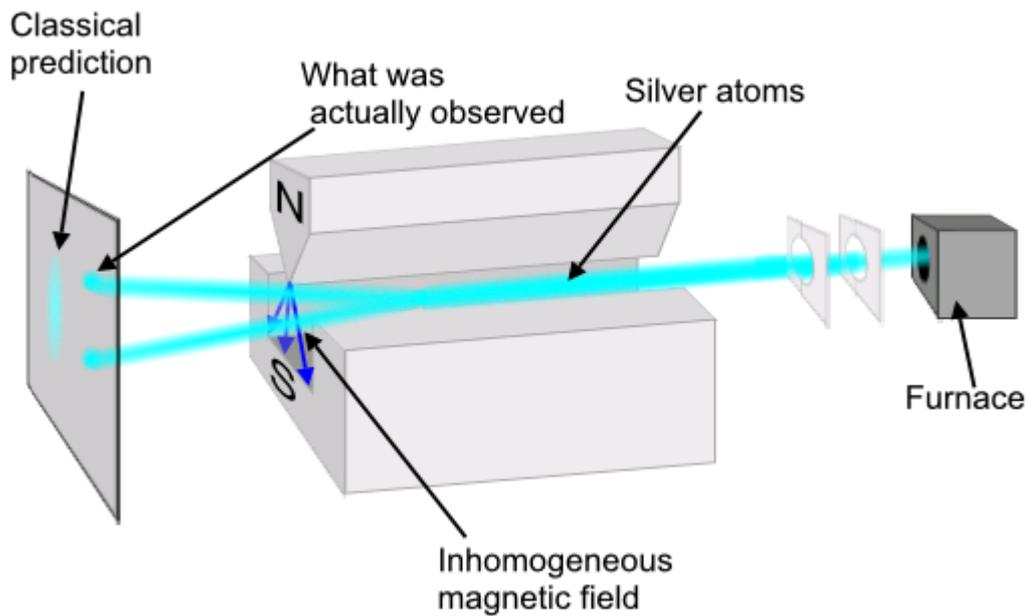


Figure 1: Stern-Gerlach experiment

The experiment uses a beam of silver atoms that passes through an inhomogeneous magnetic field. The field deflects the silver atoms based on their angular momentum orientation. Classical theory assumes a random, uniform distribution, which would result in continuous scatter. However, the result of the experiment is two points, representing spin up and spin down. This splitting (into two) makes it appropriate to view every two-level system equivalent to spin.

### 3. Qubit experiments

Instead of a bit, used by classical computers, the fundamental building block of quantum computers is a qubit. Qubits are designed to have energy states which are not equidistant from each other, which separates its ground and first excited states from higher energy levels. Thus, we can view a qubit as a two-level system, therefore it is equivalent to spin. The most basic representation of a qubit is a junction of two superconductors.

Using the IBM Q Experience platform and the Qiskit library, we can perform basic qubit measurements on real quantum chips.

#### 3.1 Resonance frequency scan

In the first measurement, we want to find the qubit resonant frequency (which corresponds to the energy gap of the first two energy levels). We will also need this frequency for further experimentation. We have a predefined estimate of the desired

frequency, so we perform a sweep around the estimated value, in search of an absorption peak. The result displays power-broadening, which means that close to resonant frequencies can also cause this effect. We will fit a Lorentzian shape (using ROOT) to obtain the central frequency as accurately as possible.

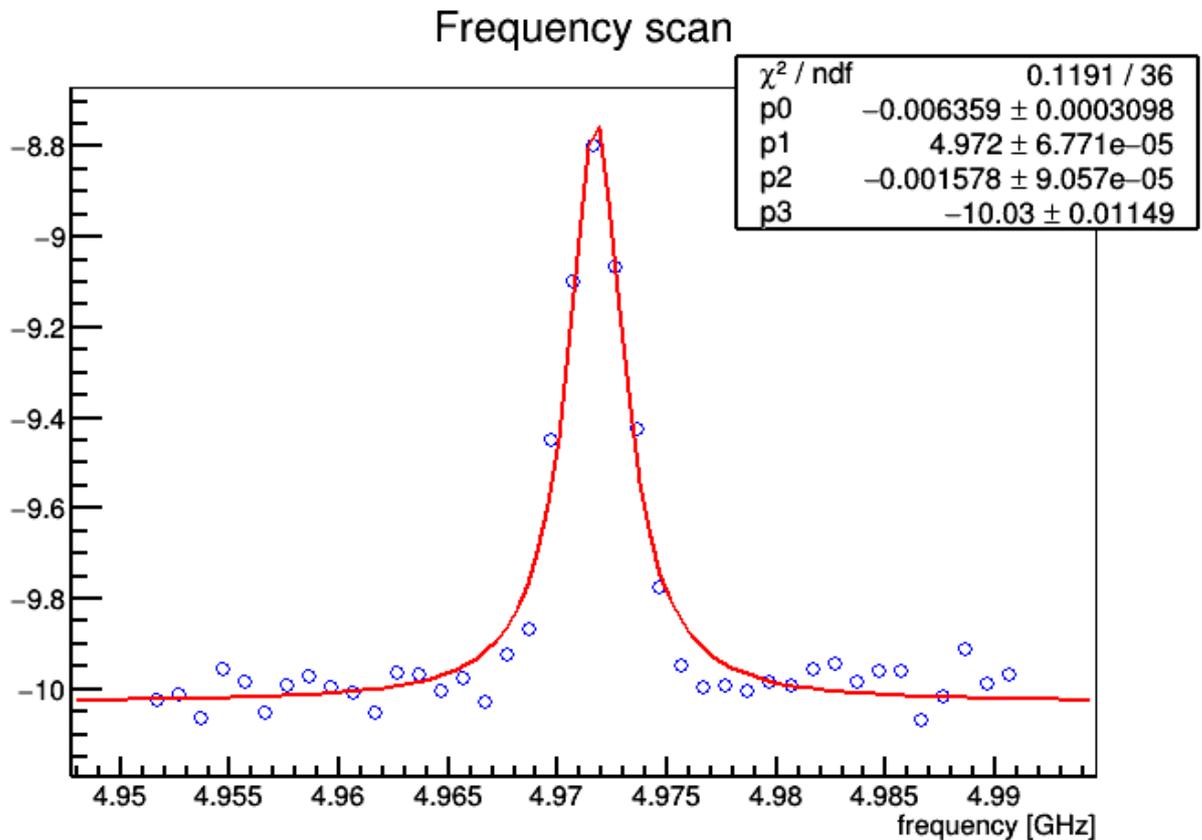


Figure 2: Frequency scan data

As the result we get  $f_0 = 4.97173 \pm 6.77092e - 05 \text{ GHz}$

### 3.2 $\pi$ Pulse Calibration

Since we obtained the frequency of the qubit we are using, we can now determine the amplitude of a  $\pi$  pulse, which excites the qubit from its ground state to higher energy and vice versa. We change the drive amplitude in increments and measure the state of the qubit after every pulse. Quantum mechanics predict that the probability of finding the qubit in its excited state will take a sinusoidal form, which is better known as Rabi oscillations. The amplitude of the  $\pi$  pulse equals to half the period of the oscillating plot.

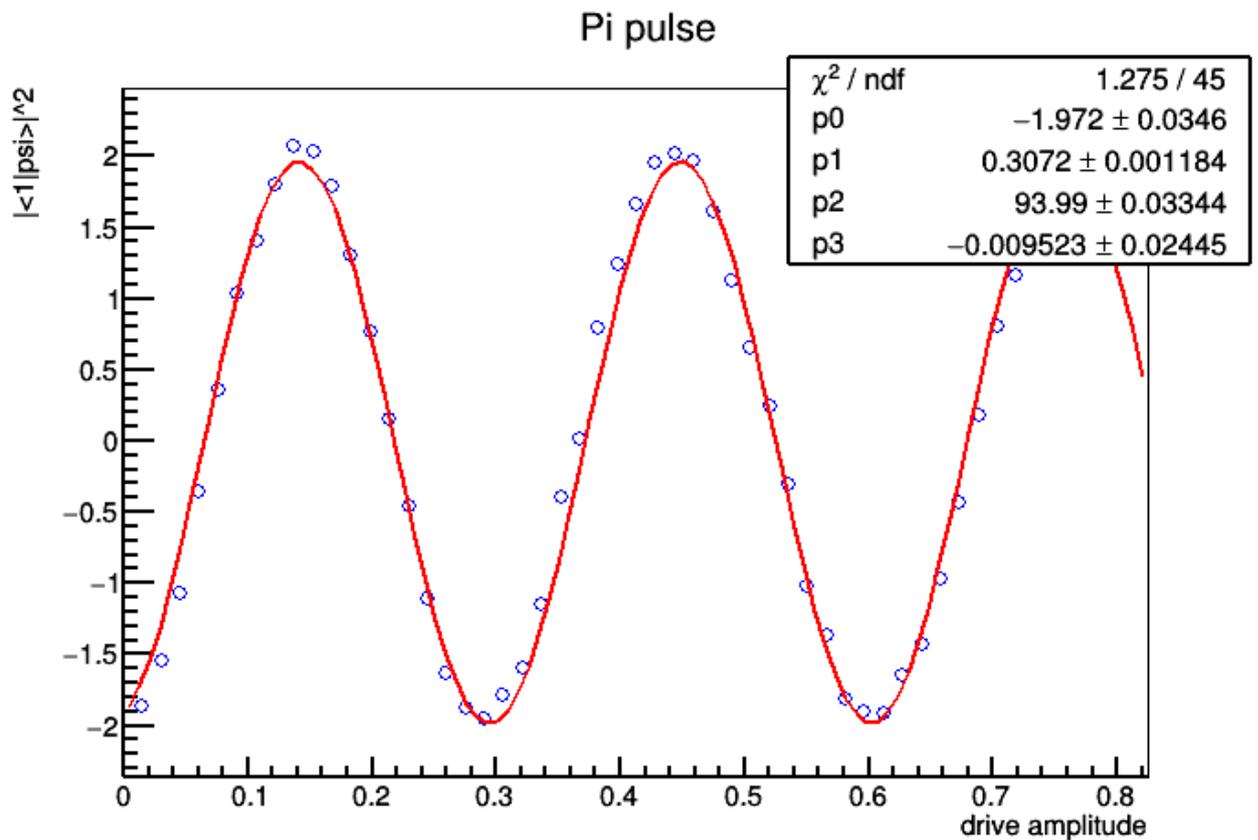


Figure 3: Rabi oscillations

We get an amplitude of 0.1535845.

### 3.3 Determining 0 vs 1

Now that we have the parameters of the pulses used for state switching, we can use the pulses to find out how do the states differ in our measurements. Our goal is to build a discriminator function which we can use in future experiments to classify qubits as 0 or 1. We plot the results in the complex plane. What we have are two clusters (blue points represent qubits in ground state and red in the excited state) which should be approximately circular. We take the average of the clusters (two black dots) and draw a line perpendicular to the line that connects the dots and is equally spaced. In the future, all experimental points that lie below this line will be considered as 0 and the ones above will represent 1.

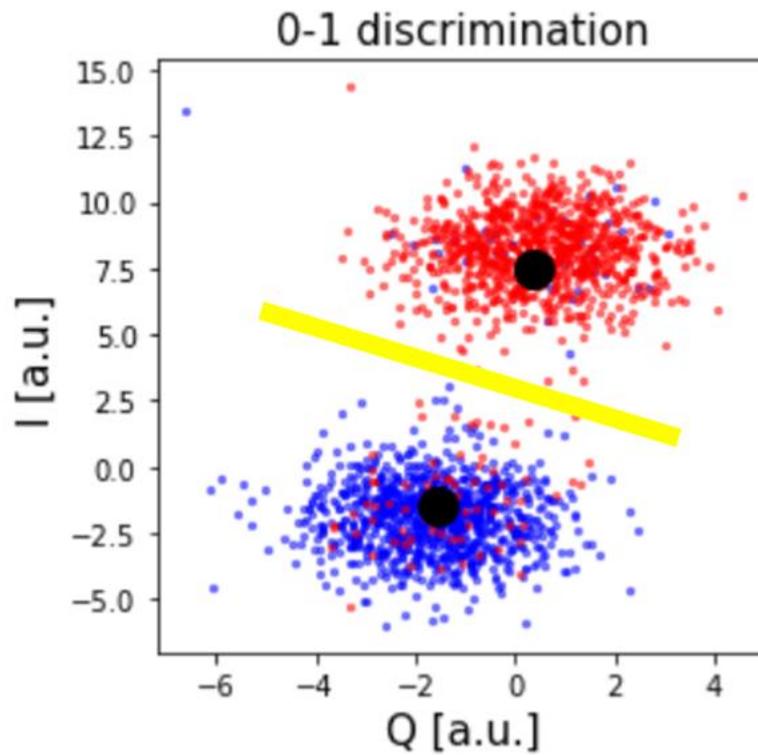


Figure 4: Frequency scan data

### 3.4 T1 Relaxation time

T1 represents the time a qubit spends in its excited state, after which it decays to the ground state. This parameter is extremely important, as it represents one of the biggest limitations of quantum computing (a qubit must not change its state on its own while computation is running). The measurement process is simple: we apply a single  $\pi$  pulse driving the qubit to its excited state, then after a delay we measure the qubits state. The goal is to vary the delay time. What we get is an exponential decay  $\sim e^{-\frac{t}{T_1}}$ . As seen on Figure 5, the decay time is estimated to be  $T_1 = 252.397 \pm 18.5588 \mu\text{s}$ .

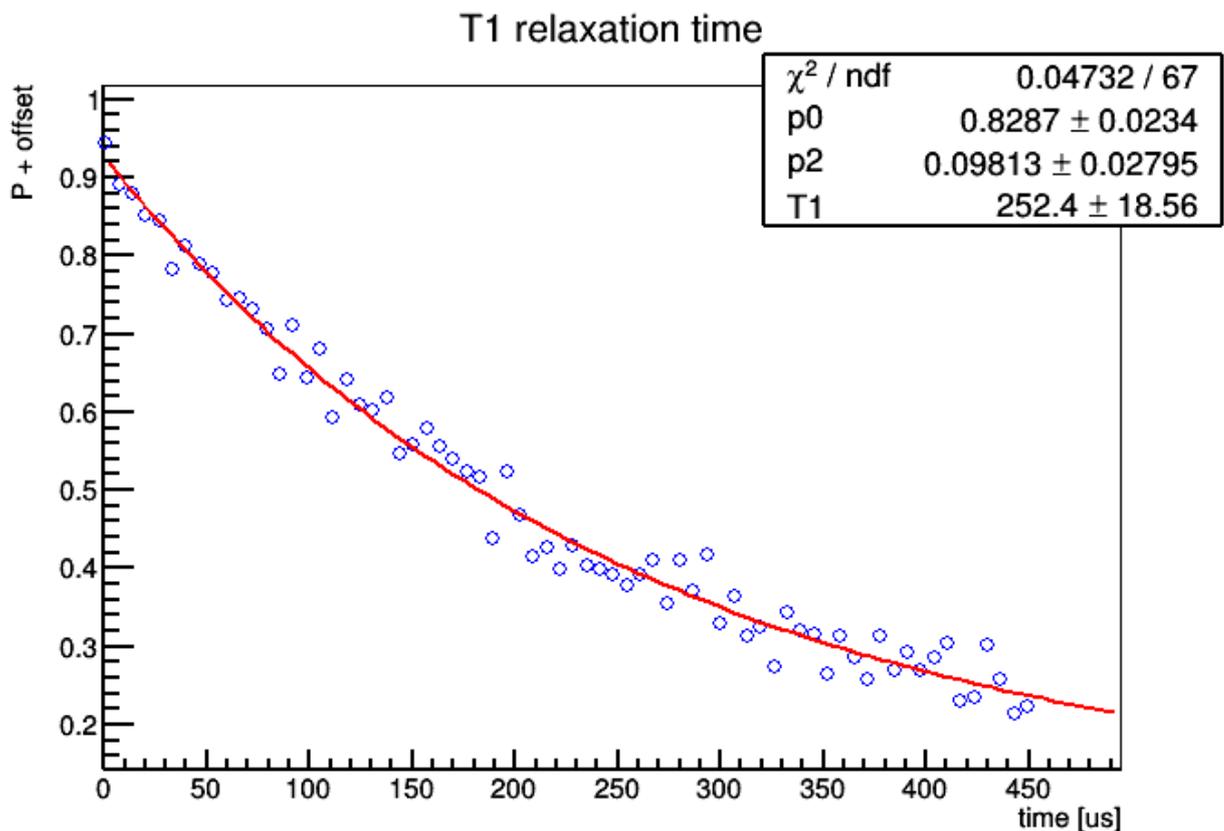


Figure 5: Relaxation time

### 3.5 Precise qubit frequency measurement using a Ramsey experiment

We wish to determine the qubit frequency with better precision, and to do so we will use a Ramsey pulse sequence. Since we are measuring the signal from the qubit at the same frequency as the pulses, we should observe oscillations at the difference in frequency between the applied pulses and the qubit. We will drive the pulses off-resonance by a known amount (frequency). The measured Ramsey signal should show oscillations with frequency near the detuning frequency, with a small offset. This small offset is exactly how far away our estimate from the first experiment was from the qubit frequency. From Figure 6 we acquire  $\text{df}=2.03581$ , which we use to calculate a more precise qubit frequency. The new updated value is  $f_0 = 4.971769 \text{ GHz}$  while it used to be  $4.97173 \text{ GHz}$ .

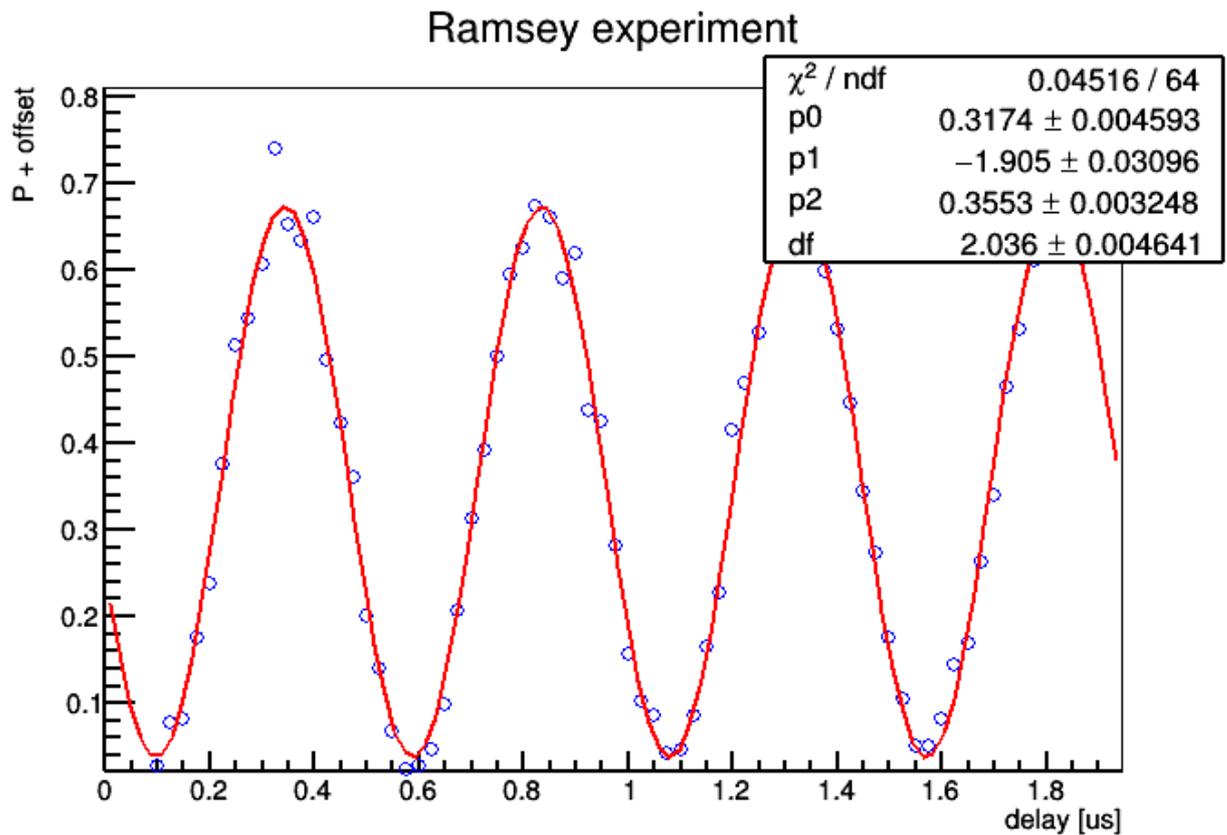


Figure 6: Ramsey experiment

### 3.6 Measuring $T_2$ using Hahn Echoes

Again we try to measure a factor of the qubit coherence ( $T_2$  time). This experiment is similar to the last one, with the addition of a  $\pi$  pulse between the two  $\pi/2$  pulses. The  $\pi$  pulse at time  $\tau$  reverses the accumulation of phase, and results in an echo at time  $2\tau$ , where we apply the last  $\pi/2$  pulse to do our measurement. The decay time for the Hahn echo experiment gives us the coherence time  $T_2$ . We get  $T_2 = 479.853 \mu\text{s}$

### 3.7 Dynamical decoupling

Improving upon Hahn echoes, we add additional  $\pi$  pulses, reducing non-quasi-static noise, therefore lengthening qubit coherence times. This is called dynamical decoupling. We get  $T_{2dd} = 286.364 \mu\text{s}$ .

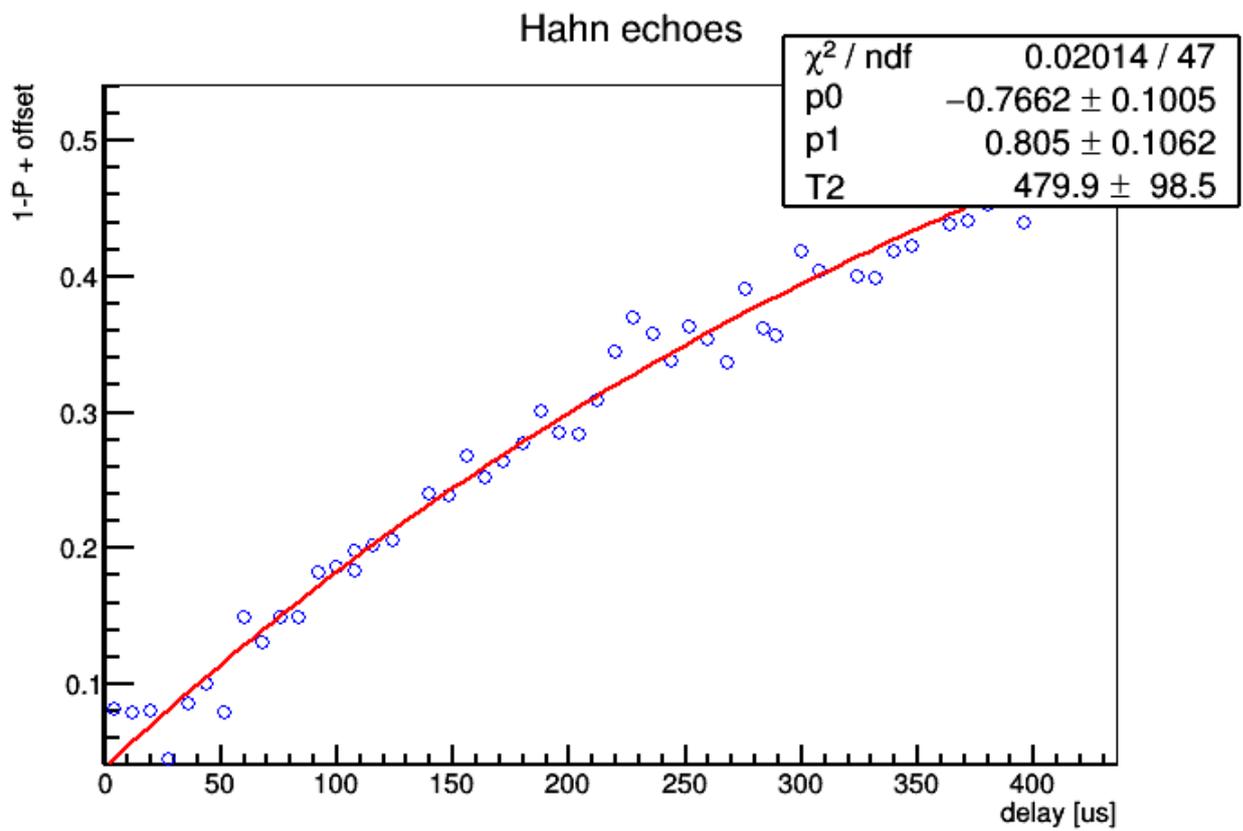


Figure 7: T2- Hahn Echoes

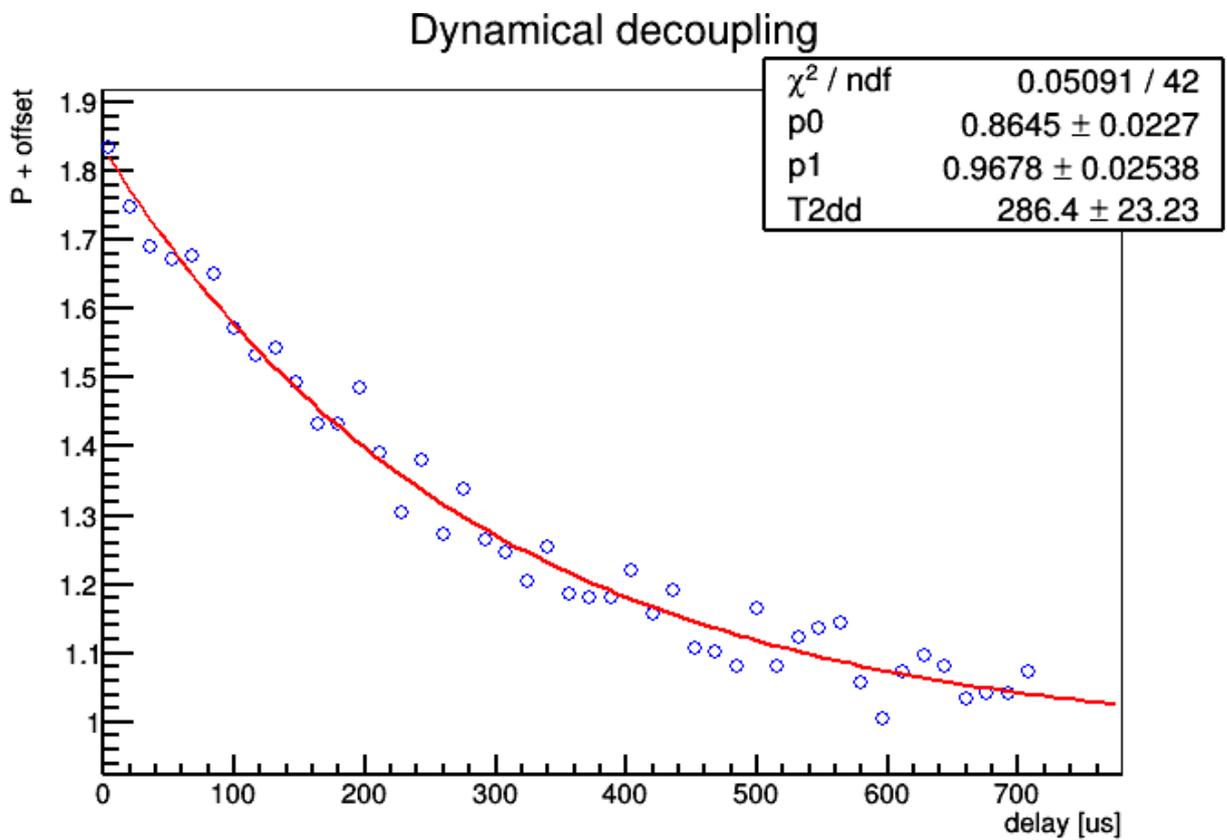


Figure 8: Dynamical decoupling

## 4. QuLogic

All operations done on a classical computer are comprised of basic logic gates. In order to run any meaningful computation on a quantum computer, we need a set of analogous quantum logic gates. What makes quantum logic gates different from classic ones is the utilization of intrinsic quantum phenomena such as superposition and entanglement. These systemic differences make it so that a quantum computer can solve a specific set of problems much faster than a classical computer ever could.

Quantum phenomena can be hard to understand, but we can simplify these mind-boggling physical occurrences by looking at them strictly mathematically (as if math has ever been simple). We view a qubits state as a simple vector with two elements. These elements represent the probability of finding the qubit in state 0, or state 1. Naturally, these two numbers have to obey normalization, as the qubit ultimately has to be in one of the two states once we measure it. These numbers can also have a phase element, which doesn't reflect itself on the probability, but since we are looking at a **wave** function, we know that phase can be of significance and used in different ways (e.g. interference). Additionally, all logic gates have their representing matrixes which we can multiply with the qubit wave function in order to see how will the qubits state change. Effectively, we can also follow the state of a qubit on a Bloch sphere, whereas the logic gates result in rotations around the Bloch sphere.

Gate	Transformation on Bloch sphere (defined for single qubit)
X	$\pi$ -rotation around the X axis, $Z \rightarrow -Z$ . Also referred to as a bit-flip.
Z	$\pi$ -rotation around the Z axis, $X \rightarrow -X$ . Also referred to as a phase-flip.
H	maps $X \rightarrow Z$ , and $Z \rightarrow X$ . This gate is required to make superpositions.
S	maps $X \rightarrow Y$ . This gate extends H to make complex superpositions. ( $\pi/2$ rotation around Z axis).
$S^\dagger$	inverse of S. maps $X \rightarrow -Y$ . ( $-\pi/2$ rotation around Z axis).
T	$\pi/4$ rotation around Z axis.
$T^\dagger$	$-\pi/4$ rotation around Z axis.

Figure 9: Bloch sphere interpretations of some single qubit logic gates

# Quantum Gates

Hadamard	$\text{---} \boxed{H} \text{---}$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$H 0\rangle = \frac{ 0\rangle +  1\rangle}{\sqrt{2}}$ $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$
Pauli-X	$\text{---} \boxed{X} \text{---}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	
Pauli-Y	$\text{---} \boxed{Y} \text{---}$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	
Pauli-Z	$\text{---} \boxed{Z} \text{---}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	
Phase	$\text{---} \boxed{S} \text{---}$	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	
$\pi/8$	$\text{---} \boxed{T} \text{---}$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	

Gate    Input    Output

Figure 10: Matrix interpretation of some single qubit logic gates

It should be noted that there are single and multiple qubit logic gates. Once we introduce multiple qubits we can start to play around with qubit entanglement.

We can use the IBM Q Experience circuit composer and QASM-2 language to play with different circuits and gain a better understanding of the underlying phenomena and its possible implementation.

## 5. Quantum algorithm

As we already mentioned, the whole purpose of quantum computers is their ability to carry out certain computations much faster than their classical counterparts can. One of the most famous examples is Shor's algorithm used for the factorization of big numbers, which is a principle used in encryption. We can demonstrate this ability with a simpler algorithm called Grover's algorithm. Grover's algorithm solves the unstructured search problem in roughly  $\sqrt{N}$  steps (where  $N$  is the number of items in the viewed list), while the speed of computation in a classical environment is  $\sim N$ . Theoretically, this algorithm should work almost 100% of the time. However, quantum computers are a work in progress. There are a lot of engineering challenges regarding hardware that have a lot of room for improvement. We mentioned one of the big problems being decoherence. This results in the algorithm working only around 60% of the time, depending on the quantum chip used. Nonetheless, quantum computing remains one of the most interesting fields in upcoming research.

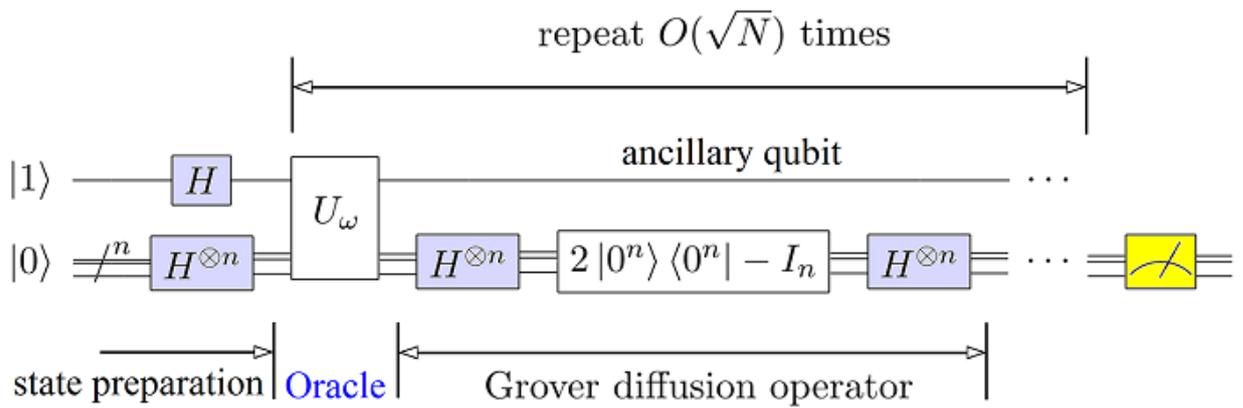


Figure 11: Grover's algorithm diagram

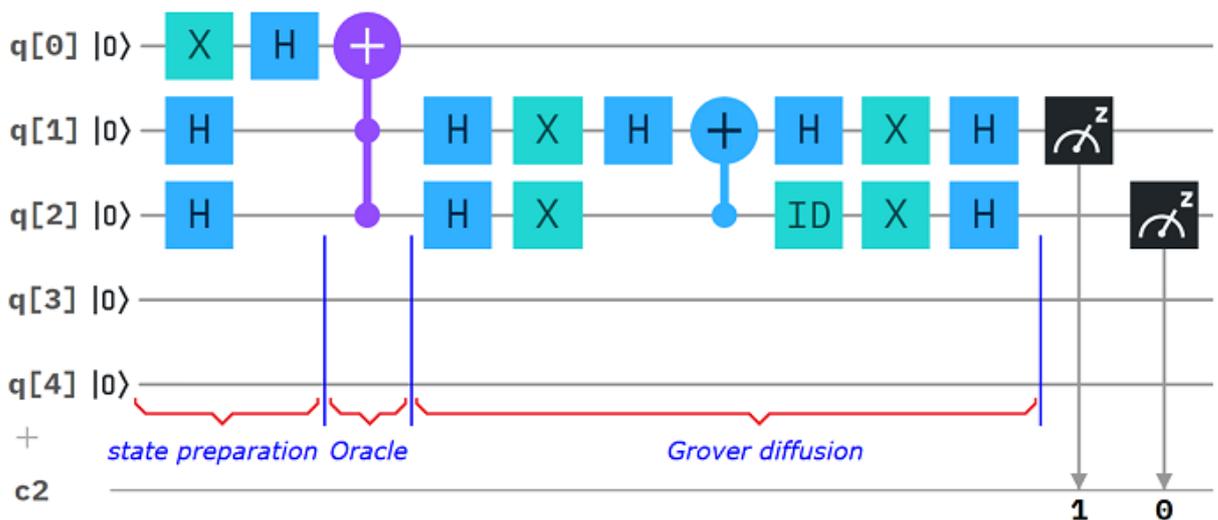


Figure 12: Grover's algorithm in IBM circuit composer

## 6. Conclusion

Throughout this program, we were introduced to the main principles of quantum mechanics and quantum computing. Through the IBM Q Experience platform, we got to work with real qubits, encounter insightful difficulties of working with real-world, non-ideal chips, but see the power this astonishing physical concept carries. The content of this course was highly interesting, at times difficult, but never insurmountable with the help of our supervisor. I would like to deeply thank the Joint Institute of Nuclear Research as well as Dr. Mihai Dima and Prof. Gheorghe Adam for the opportunity of participating in this project. I enjoyed every single lecture. As a student of Physical Electronics, this topic interests me greatly from a hardware point of view. With this program being my first real introduction into the thematic, it might push me to pursue a career in this field in the future.