Bulk properties of the medium produced in heavy ion collisions at MPD

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December 11.2020

Abstract

We have analysed Monte Carlo simulations of the transverse momentum spectrum of K^+, K^-, π^+, π^- and p^+ generated using the Ultra-relativistic Quantum Molecular Dynamics model for Au + Au collisions at centre of mass energies 4, 7, 9 and 11 GeV. We have used the blast-wave and Levy functions to fit the data in the (0.2, 1.3) and (1.09, 2.5) GeV/c p_T ranges, respectively. All fits are consistent with a kinetic freeze out temperature of the order of 200 MeV, although large χ^2s suggest spectra need to be corrected.

Contents

1 Introduction

The Standard Model of Particle Physics is the most successful theory ever developed to describe fundamental particles and their interactions. These particles [2] are described as quantum fields that abide by the rules of Quantum Mechanics and Special Relativity, and they come in two different types: fermions and bosons. Elementary fermions can be divided into 6 quarks and 6 leptons. Quarks and leptons come both in three generations of mass, with two particles in each. Hence, we divide leptons in electrons (electron, muon and tau), with electric charge -1; and neutrinos, with 0 electric charge. Similarly, we have three quarks with positive 2/3 electric charge (up, charm, top) and three quarks with negative 1/3 electric charge (down, strange, bottom). Each fermion has a corresponding antifermion with the same mass and spin but opposite electric charge. Additionally, we have 5 bosons. 4 of these are called Gauge bosons, and are the result of requiring the theory to be invariant under $U(1) \times SU(2) \times SU(3)$ Gauge transformations. The remaining, so called scalar boson is the Higgs boson: an excitation of the Higgs field, which is responsible for the mass of particles in the Standard Model.

Gauge bosons are the mediators of different interactions within the Standard Model. Photons are responsible for electromagnetism, W and Z bosons for the weak interaction, and gluons for the strong interaction. Different particles interact with these force carriers in different ways, which is represented by different charges. For instance, the electric charge is represented by a single real number, whereas the charge associated to the strong force is represented by a colour spinor: a 3D vector in colour space with basis red, green and blue. Only quarks and gluons carry colour charge, and therefore, only they participate in the strong interaction. In the same way as quarks carry colour, anti-quarks carry anti-colour. Any colour plus its anticolour combines to give white (zero colour), and the same is true for all three colours or anti-colours combined. A key property of the strong force is that gluons carry colour charge, so they can couple to each other. This means that the gluon field between two colour carriers does not scale down with distance. Instead, narrow flux tubes are created, which accumulate more energy the further apart colour carriers are pulled. Eventually, it becomes energetically favourable to produce a quark - anti-quark pair from the vacuum and form two colourless states. This is known as colour confinement and it prevents quarks from existing as free particles. Rather, they form bound, colourless states called hadrons. These colour singlet states are usually found as quark - anti-quark pairs, in which case they are called mesons; or as combinations of three quarks, which are known as baryons.

Now, the behaviour of strongly interacting matter changes dramatically at high densities (or temperatures) [4]. Consider a hadron gas. At sufficiently low densities, the system can be approximately described as a Fermi or Bose gas, as hadrons are colour singlets and any strong interactions will be residual. As we increase the density we reach the point where hadrons become tightly packed, and if we increase it further, then more and more quarks occupy the volume of a typical hadron. This leads to colour being effectively screened and, therefore, to deconfinement. Hence, we predict a phase transition in strongly interacting matter from a hadron gas to a colour conducting system known as Quark-

Gluon-Plasma (QGP), whose properties are still to be investigated. Quantum ChromoDynamics (QCD), the theory that describes the strong interaction, is a non-abelian, non-perturbative Gauge theory, which makes it extremely challenging to produce approximate, let alone exact results. An extended strategy involves discretising the theory to produce a set of equations that can be solved numerically (lattice QCD), but this approach can be applied only at the limit of zero baryon densities [6].

The Nuclotron-based Ion Collider fAcility (NICA) complex is aimed at studying the phase diagram of QCD matter through heavy ion collisions in a centre of mass energy range from 2 to 12 GeV. At sufficiently high energy, the pion exchange force that prevents nucleons from merging into each other can be overcome and nuclei wave functions overlap, producing the dense medium that we require [1]. The resulting fireball of QGP matter expands and cools down, until eventually inelastic collisions stop, followed by elastic collisions. These are the chemical and kinetic freeze outs. Once freeze-out is reached, the QGP hadronises back into bound states, which are detected. It is important to note that even though nucleons consist only of up and down quarks, interactions between partons in the QGP allow for other flavours to be introduced. Therefore, an extended strategy to explore the properties of this medium is to measure the strangeness produced in heavy ion collisions as a function of baryon density, which is related to the centrality of the collision. There are several models concerning the production of particles from a QGP, we have focused on the Blast-Wave model and the Levy function. As preparation for the MPD experiment, which will start collecting data in 2023, we have performed a fit of these functions to a set of Monte Carlo simulations of Au nuclei collisions, obtained from the Ultra relativistic Molecular Dynamics model (UrQMD), and have extracted some estimates for T_{kin}.

2 The Blast Wave Model

When studying heavy ion collisions we are often interested in the transverse momentum p_T of the resulting particles, that is the projection of their 3-momentum onto the plane transverse to the beam. We can write

$$p_T = \sqrt{p_x^2 + p_y^2}, \tag{1}$$

where the z axis is tangent to the beam. This quantity is interesting as the initial reactants have a large momentum along the beam, while any transverse momentum needs to have originated from an interaction vertex. We also need to define the invariant mass $m_T = \sqrt{m^2 + p_T^2}$, where m is the rest mass of the particle.

The blast-wave model [3] assumes all hadrons decouple simultaneously from the QGP, which gives rise to a transverse 2-dimensional blast-wave that boosts particles according to their mass. The transverse mass spectrum of particles radiated from a thermal source at temperature T is given by

$$\frac{1}{m_T}\frac{dN}{dm_T} = \frac{V}{2\pi^2} m_T K_1\left(\frac{m_T}{T}\right), \tag{2}$$

where N is the particle yield, V is the volume of the source and K_1 is the modified Bessel function of the second kind:

$$K_n = \frac{\pi}{2}\frac{I_{-n} - I_n}{\sin n\pi}, \tag{3}$$

and I_n are the Bessel functions of the first kind. We need to include some corrections to take care of transverse flow and the longitudinal expansion [5], which yields the following prediction for the p_T spectra:

$$\frac{1}{p_T}\frac{dN}{dp_T} \propto \int_0^R r dr m_T I_0\left(\frac{p_T \sinh\rho}{T_{kin}}\right) K_1\left(\frac{m_T \cosh\rho}{T_{kin}}\right), \tag{4}$$

where r is the distance to the centre of the spherical fireball and r is its radius. T_{kin} is the temperature at which kinetic freeze out occurs, and $\rho = \operatorname{arctanh}\beta_T$ is the transverse velocity profile. We assume the transverse velocity β_T (in units of c) to increase linearly from 0 at the centre to β_{max} at the surface.

3 Data analysis and code example

We have used data from a series of Monte Carlo simulations of Au + Au collisions ($\sqrt{s_{NN}} = 4, 7, 9, 11$ GeV) based on the UrQMD model to analyse the transverse momentum spectrum of pions, kaons and protons. Each data set contains one million minimum bias events for each energy. In addition to the Blast Wave model above, we have also used the Levy function from Tsallis statistics:

$$\frac{1}{2\pi p_T N}\frac{\partial^2 N}{\partial p_T \partial\eta} \propto \left(1 + \frac{p_T}{nT_{kin}}\right)^{-n}, \tag{5}$$

where n is related to the non-additivity of the thermodynamic system and η is the pseudo-rapidity of the particle. We have written a C++ code using the ROOT package to read this data from a TFile and fit it with these two functions. To do so, we have defined them as members of a fitting class and have distinguished two types of function arguments: parameters and variables. Parameters represent the quantities that we need to fit: β_{max}, T_{kin}, n and the constants of proportionality in equations (4) and (5). They are passed into the functions as an array of pointers. Variables, on the other hand, are the natural arguments of the functions, i.e. p_T in this case. This part of the code is displayed in figure 1.
It was necessary to include an offset parameter, as the spectra do not necessarily peak at zero. Once the fitting class was created, we defined the TF1 objects for the fit functions specifying the number of parameters and the range of the fit. We picked the range $(1.09, 2.5)$ GeV/c for the Levy function and $(0.2, 1.3)$ GeV/c for the Blast-Wave. This part of the code is shown in figure 2.
We also needed to specify our initial guesses for the parameters of the fits. We picked $T_{kin} \approx 0.1$ GeV, $\beta_{max} \approx 0$, $n \approx -12$. We then used the Fit method to fit the histograms in the data files with these functions. The only uncertainties considered in this project are given by the width of the histogram bins, as feed down corrections from heavier hadrons and particle identification efficiency are still in progress.

```
class fitting_class
{
public:
    Double_t Levy(Double_t* x, Double_t* pars) //The Levy function
    {
        Float_t scale = pars[0]; //scale;
        Float_t temp = pars[1]; //temperature;
        Float_t exponent = pars[2]; //exponent;
        Float_t offset = pars[3]; //offset;
        Double_t val; //output of Levy function;

        Float_t pt = x[0] + offset; //transverse momentum;

        val = pow((1 + pt / (exponent * temp)), exponent);

        return scale * val;
    }

    Double_t BlastWave(Double_t* x, Double_t* pars) //The blastwave model
    {
        Double_t scale = pars[0]; //scale;
        Double_t temp = pars[1]; //temperature;
        Double_t mass = pars[2];  //mass;

        Double_t betamax = pars[3]; //surface velocity;
        Double_t peak = pars[4]; //offset;

        Double_t pt = x[0] + peak;
        Double_t mt = TMath::Sqrt(pt * pt + mass * mass);
        Int_t    nStep = 80;
        Double_t val = 0.0;
        Double_t beta, rho;
        Double_t weight;

        for (Int_t iStep = 0; iStep < nStep; iStep++) {
            beta = TMath::Power((Double_t)(iStep + 1) / nStep, 1.0) * betamax;
            rho = TMath::ATanH(beta);
            weight = (Double_t)(iStep + 1) / nStep;
            val += weight * mt * TMath::BesselK1(mt * TMath::CosH(rho) / temp)
                * TMath::BesselI0(pt * TMath::SinH(rho) / temp);
        }

        return scale * val;
    }
};
```

Figure 1: Fitting class which contains the Levy and Blast-Wave functions. Written with Visual Studio.

```
//Define objects from fitting_class
fitting_class* MyLevy = new fitting_class();
fitting_class* MyBW = new fitting_class();

//Create fit function;
TF1* levyfit = new TF1("levyfit", MyLevy, &fitting_class::Levy, 1.09, 2.5, 4);
TF1* bwfit = new TF1("bwfit", MyBW, &fitting_class::BlastWave, 0.2, 1.3, 5);
```

Figure 2: Definition of TF1 objects for the fit. Written with Visual Studio.

4 Results

We discuss the raw transverse momentum spectra of protons, π^+, π^-, K^+ and K^-. The fitted histograms at $\sqrt{s_{NN}} = 4$ GeV are shown in figures 3, 4 and 5.

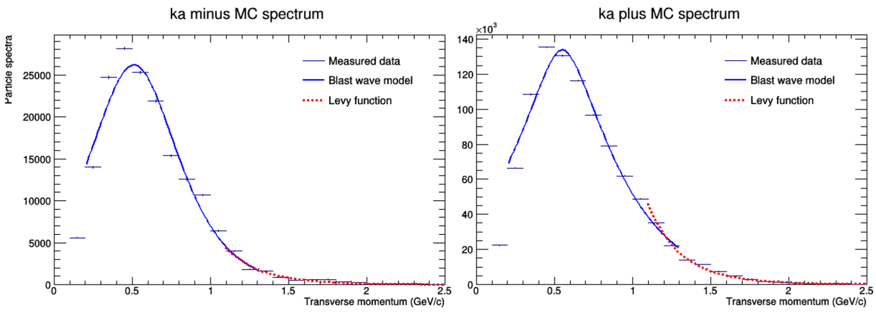

Figure 3: Kaon transverse momentum spectra at $\sqrt{s_{NN}} = 4$ GeV. The reduced χ^2 is 62.5 for the Levy function and 353 for the Blastwave-Model on the left, and 102 and 876 on the right.

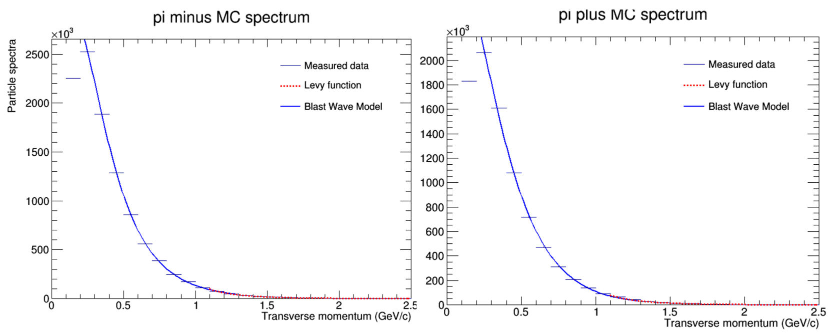

Figure 4: Pion transverse momentum spectra at $\sqrt{s_{NN}} = 4$ GeV. The reduced χ^2 is 217 for the Levy function and 427 for the Blastwave-Model on the left, and 202 and 1077 on the right.

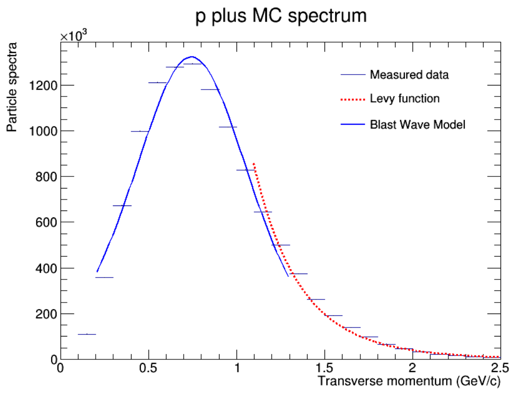

Figure 5: Proton transverse momentum spectrum at $\sqrt{s_{NN}} = 4$ GeV. The reduced χ^2 is 3830 for the Levy function and 12271 for the blast-wave model.

The results are summarised in tables 1 and 2, which show the parameters yielded by the blast-wave and Levy fits, respectively.

Results from blast-wave fit at $\sqrt{s_{NN}} = 4$ GeV							
Particle name	χ^2	Scale	T_{kin} (GeV)	Mass (GeV)	$\beta_{max}(c)$	offset (GeV/c)	Degrees of freedom
K^+	5254	1.33×10^4	0.269	7.57×10^{-2}	6.76×10^{-2}	-0.55	6
K^-	2119	1.02×10^4	0.175	0.293	9.28×10^{-2}	-0.51	6
π^+	6462	2.90×10^5	0.202	0.01	0.157	-0.171	6
π^-	2564	4.06×10^5	0.201	1.33×10^{-2}	0.251	-0.132	6
p^+	73623	8.04×10^5	0.186	0.421	1.08×10^{-3}	-0.741	6

Table 1: Results from blast-wave fit at 4 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra. The masses of the particles are not consistent with accepted values.

Results from Levy fit at $\sqrt{s_{NN}} = 4$ GeV						
Particle name	χ^2	Scale	T_{kin} (GeV)	n	offset (GeV/c)	degrees of freedom
K^+	1024	1.01×10^4	0.242	-12	4.37	10
K^-	625	1.78×10^4	0.233	-12	4.89	10
π^+	2016	2.70×10^4	0.211	-12	3.74	10
π^-	2166	5.18×10^3	0.257	-12	4.41	10
p^+	38303	1.73×10^6	0.270	-12	5.60	10

Table 2: Results from Levy fit at 4 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra.

The fitted histograms for $\sqrt{s_{NN}} = 7$ GeV are shown in figures 6, 7 and 8; and their respective fitted parameters can be found in tables 3 and 4.

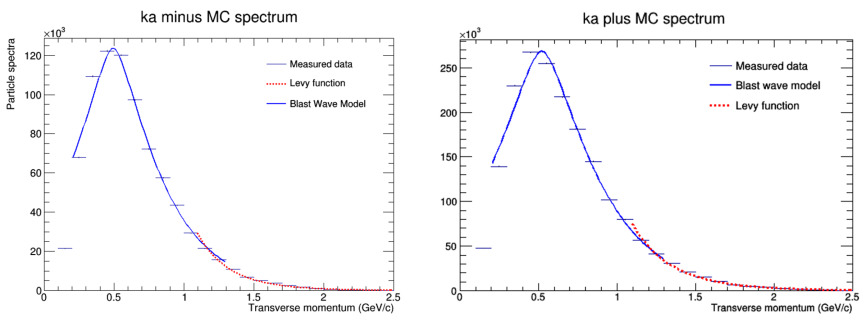

Figure 6: Kaon transverse momentum spectra at $\sqrt{s_{NN}} = 7$ GeV. The reduced χ^2 is 129 for the Levy function and 356 for the blast-wave model on the left, and 172 and 1340 on the right.

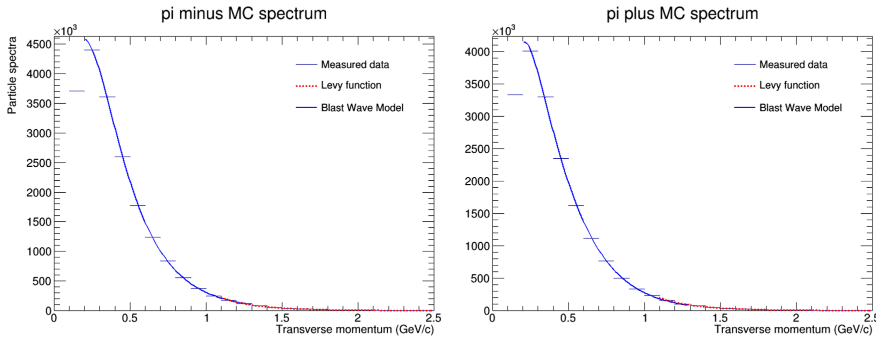

Figure 7: Pion transverse momentum spectra at $\sqrt{s_{NN}} = 7$ GeV. The reduced χ^2 is 50.9 for the Levy function and 880 for the blast-wave model on the left, and 134 and 128 on the right.

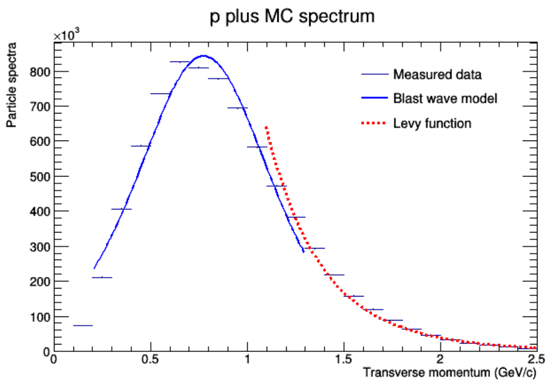

Figure 8: Proton transverse momentum spectrum at $\sqrt{s_{NN}} = 7$ GeV. The reduced χ^2 is 1823 for the Levy function and 8243 for the blast-wave model.

Results from Levy fit at $\sqrt{s_{NN}} = 7$ GeV						
Particle name	χ^2	Scale	T_{kin} (GeV)	n	offset (GeV/c)	Degrees of freedom
K^+	1724	1.57×10^5	0.232	-12	4.69	10
K^-	1290	3.40×10^6	0.141	-12	3.11	10
π^+	1344	1.99×10^3	0.340	-12	5.77	10
π^-	509	7.66×10^5	0.202	-12	4.00	10
p^+	18230	1.20×10^6	0.270	-12	5.56	10

Table 3: Results from Levy fit at 7 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra.

Results from blast-wave fit at $\sqrt{s_{NN}} = 7$ GeV							
Particle name	χ^2	Scale	T_{kin} (GeV)	Mass (GeV)	$\beta_{max}(c)$	offset (GeV/c)	Degrees of freedom
K^+	8042	2.59×10^4	0.264	3.39×10^{-2}	0.219	-0.52	6
K^-	2138	1.94×10^4	0.218	8.89×10^{-2}	0.667	-0.499	6
π^+	768	5.38×10^5	0.196	1.98×10^{-2}	0.427	-0.247	6
π^-	5277	5.28×10^5	0.212	1.07×10^{-2}	3.52×10^{-2}	-0.221	6
p^+	49459	8.41×10^5	0.177	0.501	1.57×10^{-2}	-0.775	6

Table 4: Results from blast-wave fit at 7 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra. The masses of the particles are not consistent with accepted values.

Fitted histograms for $\sqrt{s_{NN}} = 9$ GeV can be found in figures 9, 10 and 11; and theiir corresponding fitted parameters are displayed in tables 5 and 6.

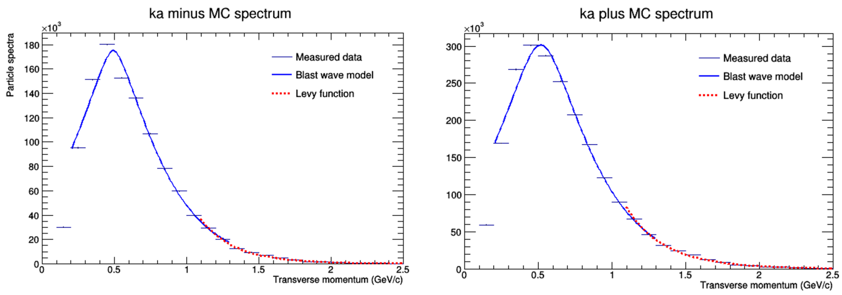

Figure 9: Kaon transverse momentum spectra at $\sqrt{s_{NN}} = 9$ GeV. The reduced χ^2 is 46.5 for the Levy function and 780 for the blast-wave model on the left, and 93.4 and 1152 on the right.

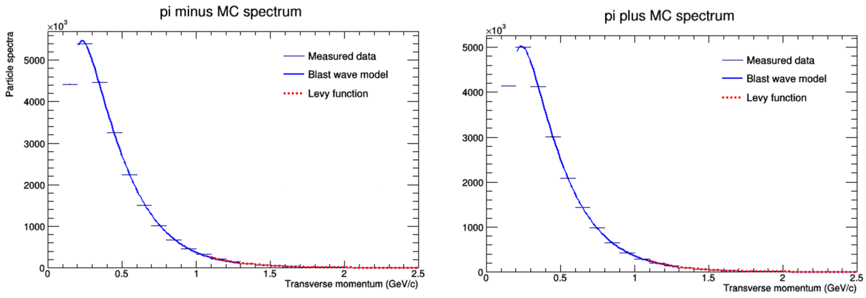

Figure 10: Pion transverse momentum spectra at $\sqrt{s_{NN}} = 9$ GeV. The reduced χ^2 is 85.3 for the Levy function and 1477 for the blast-wave model on the left, and 138 and 157 on the right.

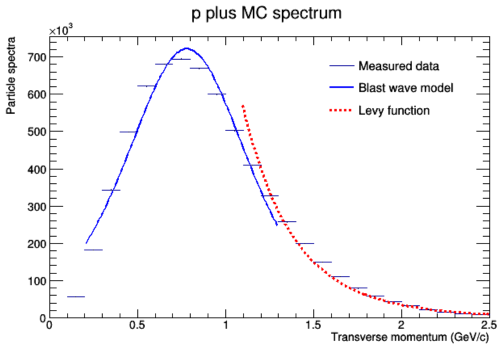

Figure 11: Proton transverse momentum spectrum at $\sqrt{s_{NN}} = 9$ GeV. The reduced χ^2 is 2275 for the Levy function and 6108 for the blast-wave model.

Results from Levy fit at $\sqrt{s_{NN}} = 9$ GeV						
Particle name	Reduced χ^2	Scale	T_{kin} (GeV)	n	offset (GeV/c)	Degrees of freedom
K^+	934	2.42×10^3	0.348	-12	6.18	10
K^-	465	1.21×10^4	0.259	-12	4.83	10
π^+	1379	2.04×10^3	0.344	-12	5.79	10
π^-	853	3.17×10^3	0.346	-12	5.92	10
p^+	22746	1.13×10^6	0.270	-12	5.57	10

Table 5: Results from Levy fit at 9 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra.

Results from blast-wave fit at $\sqrt{s_{NN}} = 9$ GeV							
Particle name	χ^2	Scale	T_{kin} (GeV)	Mass (GeV)	$\beta_{max}(c)$	offset (GeV/c)	Degrees of freedom
K^+	6911	3.12×10^4	0.263	8.67×10^{-2}	6.23×10^{-4}	-0.518	6
K^-	4679	1.74×10^4	0.250	1.09×10^{-2}	4.06×10^{-5}	-0.493	6
π^+	943	6.87×10^5	0.200	4.75×10^{-2}	0.374	-0.238	6
π^-	8863	6.47×10^5	0.210	1.00×10^{-2}	3.58×10^{-2}	-0.233	6
p^+	36649	4.93×10^6	0.187	0.459	1.64×10^{-3}	-0.775	6

Table 6: Results from blast-wave fit at 9 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra. The masses of the particles are not consistent with accepted values.

The fitted histograms for $\sqrt{s_{NN}} = 11$ GeV are shown in figures 12, 13 and 14; and the corresponding fitted parameters are displayed in tables 7 and 8.

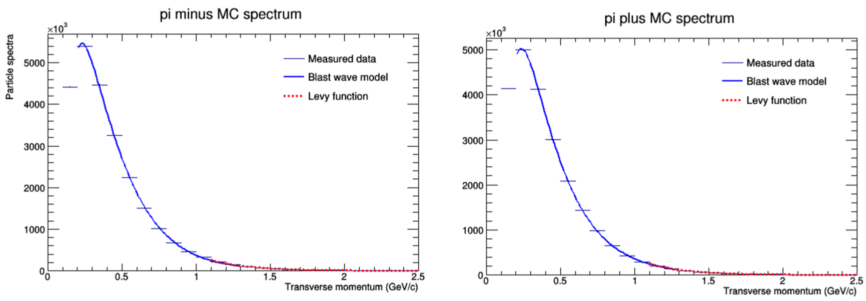

Figure 12: Kaon transverse momentum spectra at $\sqrt{s_{NN}} = 11$ GeV. The reduced χ^2 is 173 for the Levy function and 351 for the blast-wave model on the left, and 70.9 and 279 on the right.

Results from Levy fit at $\sqrt{s_{NN}} = 11$ GeV						
Particle name	χ^2	Scale	T_{kin} (GeV)	n	offset (GeV/c)	Degrees of freedom
K^+	1489	2.67×10^5	0.249	-12	5.18	10
K^-	1729	1.22×10^3	0.323	-12	5.65	10
π^+	944	1.94×10^5	0.253	-12	4.87	10
π^-	410	1.90×10^3	0.359	-12	6.02	10
p^+	126401	3.70×10^3	0.333	-12	5.52	10

Table 7: Results from Levy fit at 11 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra.

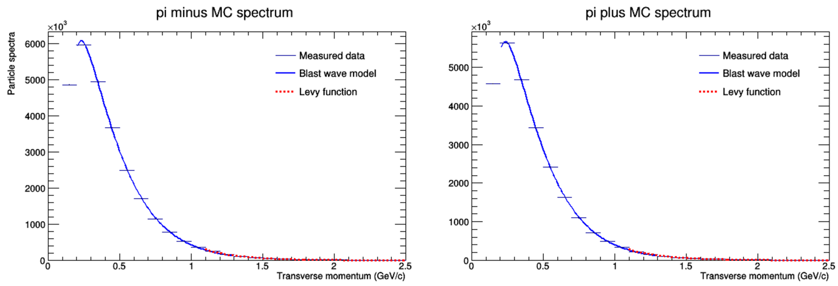

Figure 13: Pion transverse momentum spectra at $\sqrt{s_{NN}} = 11$ GeV. The reduced χ^2 is 41 for the Levy function and 1363 for the blast-wave model on the left, and 94.4 and 817 on the right.

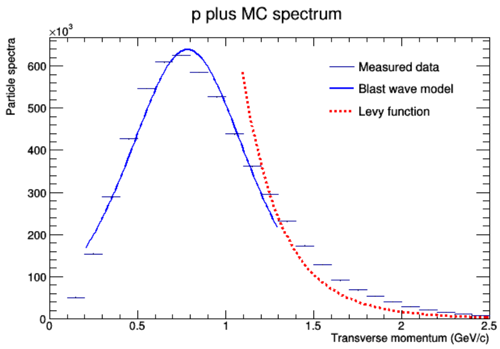

Figure 14: Proton transverse momentum spectrum at $\sqrt{s_{NN}} = 11$ GeV. The reduced χ^2 is 12640 for the Levy function and 6020 for the blast-wave model.

Results from blast-wave fit at $\sqrt{s_{NN}} = 11$ GeV							
Particle name	χ^2	Scale	T_{kin} (GeV)	Mass (GeV)	$\beta_{max}(c)$	offset (GeV/c)	Degrees of freedom
K^+	5579	4.03×10^4	0.247	0.131	5.66×10^{-5}	-0.515	6
K^-	2103	3.18×10^4	0.220	0.130	0.480	-0.504	6
π^+	4899	6.69×10^5	0.211	1.70×10^{-2}	1.52×10^{-3}	-0.237	6
π^-	8177	7.19×10^5	0.211	1.02×10^{-2}	0.146	-0.234	6
p^+	36120	4.69×10^5	0.183	0.458	7.31×10^{-4}	-0.783	6

Table 8: Results from blast-wave fit at 11 GeV. Large reduced χ^2 are due to the fact that we have analysed raw spectra. The masses of the particles are not consistent with accepted values.

5 Conclusion

To conclude, we have analysed the different particle transverse momentum spectra produced by Monte Carlo simulations based on UrQMD for $\sqrt{s_{NN}} = 4, 7, 9, 11$ GeV. We have used two models: the blast-wave model and the Levy function, from relativistic fluid dynamics and Tsallis statistics respectively. We fitted the particle spectra in the shape of histograms with C++ ROOT in two different momentum ranges: (0.2, 1.3) GeV/c for the blast-wave and (1.09, 2.5) GeV/c for the Levy function. Both models suggest an order of magnitude for the freeze out temperature of 200 MeV, although the large χ^2 of the fits imply the data needs to be further processed and the different feed down corrections taken into account.

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